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# THE MATHEMATICS TEACHER

Volume XLV · OCTOBER · 1952 · Number 6

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# THE MATHEMATICS TEACHER

**Official Journal of the National Council of Teachers of Mathematics**

*Devoted to the interests of mathematics teachers in Elementary and Secondary Schools, Junior Colleges and Teacher Education*

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*Associate Editors and Additional Members of the Committee on Official Journal*—Phillip S. Jones, University of Michigan, Ann Arbor, Michigan; Henry W. Syer, Boston University, Boston, Massachusetts; Edith Wooley, Sanford Junior High School, Minneapolis, Minnesota.

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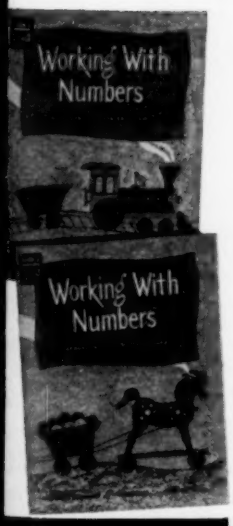


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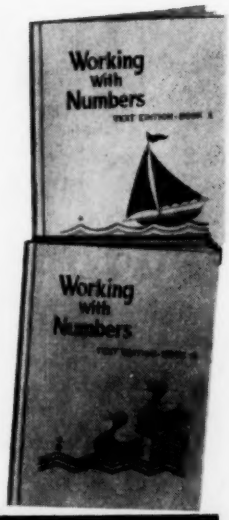


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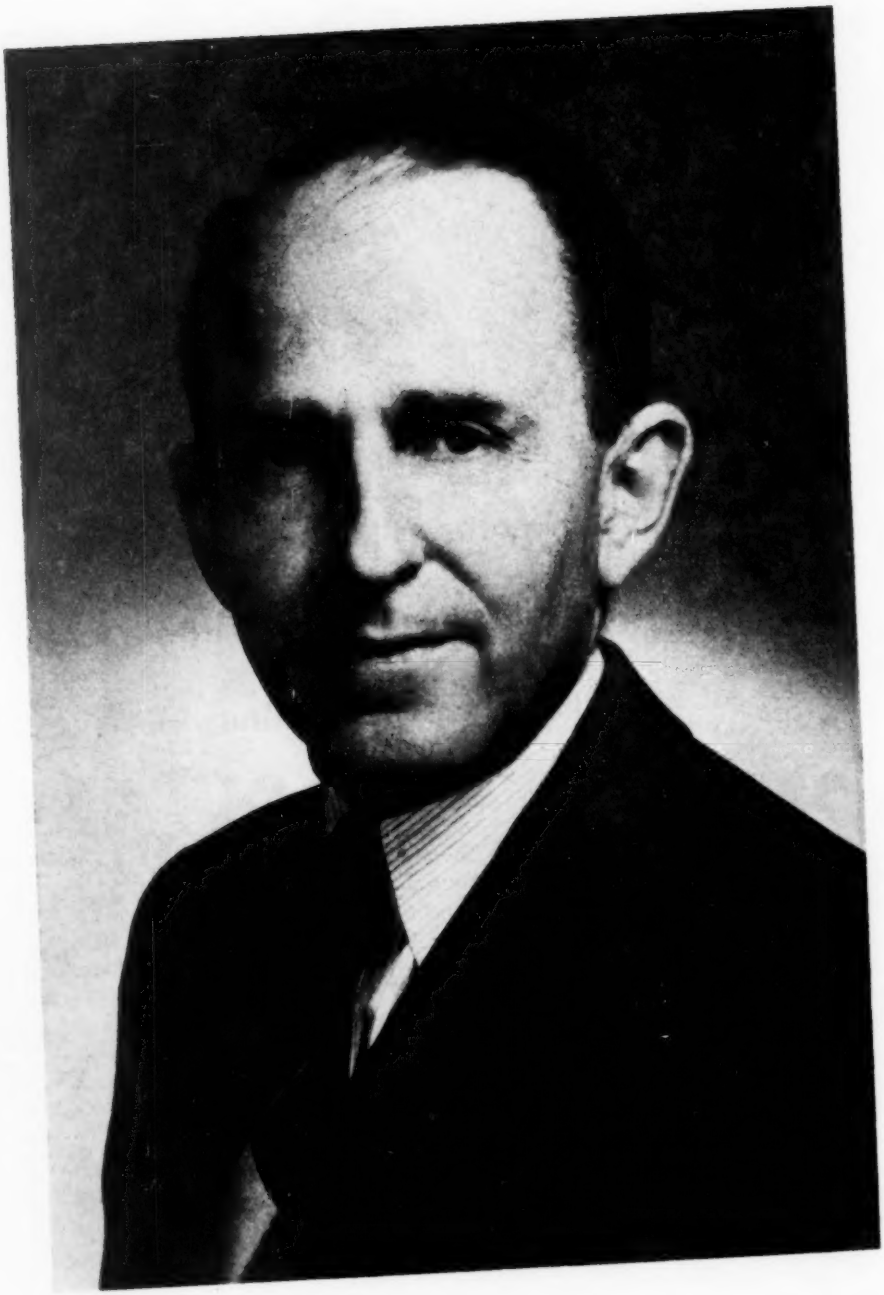

JOHN R. MAYOR
President
The National Council of Teachers of Mathematics
1952–

# THE MATHEMATICS TEACHER

Volume XLV Number 6

October 1952

## The President's Page

THE Thirtieth Annual Meeting in Des Moines in April, 1952 in many ways revealed the present and potential strength of The National Council of Teachers of Mathematics. The nearly one thousand persons in attendance heard one of the best programs ever offered by our organization. The fine spirit of the membership was revealed by the enthusiastic response of those in the various sessions; the grand hospitality of the host organization, the Iowa Association of Mathematics Teachers; and the genuine interest in the welfare of the Council demonstrated by the official Delegates at the Delegate Assembly and by the large number of members who attended the annual business meeting. This is only a part of the evidence of what our organization can do and has to offer.

In this year of 1952 when the teaching of mathematics at all levels of instruction is more important than it has ever been before, it is a source of satisfaction to all members to know that the National Council is in a position to contribute greatly to the improvement of American schools. Our principal regret is that the services of the National Council are not reaching more mathematics teachers everywhere. Let us resolve that another year will see the influence of the National Council improve and grow! We all bear a responsibility to see that our colleagues and our communities become more familiar with the services of the only organization devoted to the improvement of mathematics teaching at all levels of instruction.

One of the ways in which the influence of our organization can be strenthened is through increased membership. Too, an increase in membership, bringing about an increase in funds for the program of the National Council, is essential if we are to carry out our present activities and improve and increase them. While our journal is already one of the best, if not the best, of any subject matter professional organization, and new publications are being planned and will soon be available to teachers, a large number of excellent ideas for services of the National Council were presented to the Board of Directors in Des Moines for which no money in the present budget is available. An increase in our membership will make these services possible and at the same time insure that our present fine professional materials reach more teachers.

Our recent affiliation with the National Education Association and the very presence of our national office in the N.E.A. building in Washington, open new opportunities for co-operation with other professional groups and to influence school programs on a national scale. The fifty-seven Affiliated Groups are constantly working with state, county, and city education associations in all parts of the country. Individual members of the Council are inspired, through their association in a national organization with other teachers of similar ideals and goals, to assume roles of leadership in education associations, state committees, and school

projects. These instances of co-operation illustrate means of meeting a very important goal of the National Council and must be encouraged in every way.

The work and planning, that make the success of a professional organization even on a national scale, so often fall on the shoulders of too small a group who are willing or who are fortunate enough to be chosen. In order for The National Council of Teachers of Mathematics to achieve its maximum effectiveness there must be much broader participation in the planning and the implementation of the activities of the Council. There is great talent among our members and it is my resolution to make certain, with the help and guidance of all members, that many more people have an active part in our organization than has ever been true before. Thousands of mathematics teachers working together for the worthy cause, which is ours, can make a truly significant contribution to a stronger America and a better world.

JOHN R. MAYOR
*President*

---

## MINUTES OF ANNUAL BUSINESS MEETING

### National Council of Teachers of Mathematics
### Hotel Fort Des Moines, Des Moines, Iowa
### Saturday, April 19, 1952

The meeting was called to order by the President, Harry W. Charlesworth.

The following motion was made by Henry Swain and seconded by R. R. Smith:

THAT we dispense with the reading of the minutes of the last meeting at Pittsburgh, April, 1951, because they had been published in THE MATHEMATICS TEACHER for October, 1951. Motion carried.

H. W. Charlesworth introduced H. Vernon Price who presented a report on the Reorganization of Secondary Schools in Iowa emphasizing the special problems of the small high school. Discussion followed.

Henry Syer requested that materials useful to small high schools be sent to him with suggestions for use in small publications.

H. C. Christofferson suggested that a section of the program at future meetings, perhaps starting with the Christmas 1952 meeting in Nebraska, be set aside for discussion of small school problems.

Notion by John R. Mayor and seconded by Miss Veryl Schult:

THAT the paper of H. Vernon Price be incorporated in the minutes. Motion carried. (See Appendix attached.)

Motion by O. E. Overn and seconded by Miss Sward:

THAT the Chair appoint a committee of three to consider the problem of what the National Council of Teachers of Mathematics can do for the small high school, and report at the next meeting. This motion was superceded by a substitute motion.

Substitute motion by H. C. Christofferson and seconded by Miss Sward:

THAT the problem of the small high school be referred to the Program Committee with the recommendation that they consider it for the Nebraska Christmas meeting. Substitute motion carried.

M. H. Ahrendt, Executive Secretary, was introduced. He first paid tribute to President Charlesworth and expressed a deep sense of gratitude for the help and understanding given him in carrying out his duties in the Washington office. He then reported on the Regional Conferences of Commission on Teacher Education and Professional Standards. Thirdly, he gave the list of the delegates of the eight conferences as follows: (Several of these persons represented organizations other than the National Council) Henry W. Syer, M. L. Herman, Veryl Schult, Daniel B. Lloyd, F. Lynwood Wren, Kenneth E. Brown, James H. Zant, Eunice Lewis, H. Van Engen, O. A. George, Dale Carpenter, Elenore M. Lazansky, Sylvia Vopni, Helen Dunn, Lenore John, George E. Hawkins, Agnes Herbert, and Dorothy Meagher.

Mr. Ahrendt then stated that at a meeting held Friday morning, April 18th, the present Des Moines delegates met to discuss the results of the conferences and to consider whether any outcomes of the conferences should be presented to the members of the Council. The following situations were pointed out:

1. The members of the Council have for many years been discussing the need for improvement in the professional stand-

ards and training of mathematics teachers.

2. Many groups over the nation are discussing the general problems of teacher education. Most of this discussion ignores or misunderstands the needs of mathematics education.
3. Many schools and state departments of education are seeking information on the preparation needed for the training and certification of mathematics teachers.

In order to meet the above situations, it was recommended that the following lines of action should be followed by the Council:

1. A study be made to determine the competencies needed by a teacher for the teaching of mathematics. Some years ago we approved a list of 29 criteria for functional competency on the part of students. The competencies needed to teach mathematics should likewise be investigated.
2. The experiences needed to secure these competencies should also be determined. These experiences should not necessarily be defined in terms of semester hours. They should include the planning of new courses, the revision of old courses, the planning of laboratories, and the like, with emphasis on both subject matter and the teaching of mathematics.

Discussion followed.

Mr. Rosskopf suggested the above named group participate in the Symposium at Madison, Wisconsin. Mr Hawkins expressed the need for getting suggestions for certification.

Motion by M. F. Rosskopf and seconded by H. C. Christofferson:

THAT the question of certification of secondary mathematics teachers be referred to the Symposium on Teacher Education in Mathematics, Madison, Wisconsin, August 26–30.

John R. Mayor related the Symposium suggested mathematics courses for the secondary teachers but thought they may compromise and include elementary. Motion carried.

Motion by H. C. Christofferson and seconded by M. F. Rosskopf:

THAT the whole problem of certification of both elementary and secondary schools be referred to the Board of Directors for study and action. Motion carried.

Houston Karnes presented the Declaration of Principles and pointed up thoughts in behalf of a Platform. Summary of the thinking was that the Platform should be no one individual's thinking but rather needed to be the work of a group. Three aspects pointed out were:

1. Let educational public in general know our stand on things mathematical.
2. Stand as a guide for all teachers of mathematics as we face our daily problems.
3. Serve as an instrument of public relations to laymen.

Discussion followed.

Motion by R. R. Smith and seconded by H. C. Christofferson:

THAT the matter of a Platform as outlined by Houston Karnes be referred to the Board as one of its important considerations. Motion carried.

H. W. Charlesworth announced it was time to turn over the Presidency to the newly elected member, but that he was sorry no gavel was at hand to turn over.

Veryl Schult arose to the occasion and quickly went forward to hand H. W. Charlesworth a gavel which the Wisconsin members wished to have presented to John R. Mayor in accepting the office of President for the next two years. Dr. Mayor received the gavel amid a loud applause.

He thanked the group for their expression of confidence and stated he felt humble and frightened with the difficult problems ahead. The following things were mentioned by John R. Mayor as things he would like to accomplish:

1. Fine services of the N.C.T.M. to reach more people.
2. See work of the Council spread among more people.
3. Improve the many fine services of the Council, namely:
   a. THE MATHEMATICS TEACHER
   b. The Monographs
   c. Special Publications
   d. Research
   e. Programs
4. Do everything we can to see that our Washington office shows a new influence on a national scale.
5. Increase cooperation between mathematics groups and other professional groups in all parts of the country.
6. Improve our schools for the youth of our country.

Ballot returns show the following new officers were elected:

President (for two years): John R. Mayor
Vice-President Senior High School (for two years): Marie Wilcox
Vice-President Junior High School (for one year): Agnes Herbert
Vice-President Elementary School (for two years): Irene Sauble
Three members to Board of Directors (for three years): Allene Archer, Ida May Bernhard, Harold Fawcett

Motion by H. C. Christofferson and seconded by R. R. Smith:

THAT an expression of thanks be given to H. W. Charlesworth for his fine work of the past years to the Council. Motion carried.

The meeting was then declared adjourned.

Respectfully submitted
AGNES HERBERT, *Recording Secretary*

# The Small High School

*By* H. VERNON PRICE
*State University of Iowa, Iowa City, Iowa*

It is only natural for a teacher in the Midwest to be interested in the small high school and to recognize that a school of this type has special problems of its own. My personal interest in these problems was heightened by our experiences in re-writing the Iowa Course of Study in Secondary School Mathematics a few years ago. We soon discovered that one of our major tasks was to make provision for the needs of the small schools; indeed, it is essentially true that the course of study was written for these schools.

The only recognition the National Council has given to the small schools, so far as I can determine, is in the Second Report of the Commission on Post-War Plans. One experiment was quoted in that report to support the suggestions which were made but, judging from the context, the report was presumably written from a logical, rather than an experimental, point of view. That is, one is obliged to wonder whether the suggestions made were merely theory or had some basis in practice. Do we really *know* what the problems of the small school are?

### *What is a small school?*

Perhaps we should go further and ask what we mean by a small school. Witness the following possibilities:

1. In the Commission report already cited, the following statements are made: "More than two-thirds of all high schools are small, with certainly fewer than 200 students and probably fewer than 8 teachers. Such high schools in all enroll more than a million pupils."
2. A few paragraphs later in the same report we find, "Let us assume that we are dealing with a school that has only six pupils in each grade from 9 to 12 inclusive."
3. At the 1949 Annual meeting of the Council in Baltimore, I conducted a discussion group entitled "The Mathematics Curriculum in the Small High School." Five people attended and the schools represented ranged in size from 250 to 800.
4. Our ideas in Iowa are somewhat different. The median-sized high school in this state is approximately 67.
5. A report stemming from the U. S. Office of Education in 1946 gives the following information:
   a. The median-sized 4-year high school contains about 110–115 pupils.
   b. The median-sized high school (all kinds) contains slightly less than 100 pupils.

Here, then, are several possible definitions of a small high school. In order to be definite suppose we refer to the report of the Office of Education and assume that a small high school is one which consists of less than 115 pupils.

### *Are these schools important to the National Council?*

The teacher in the small school is probably: 1. inexperienced, 2. getting little, if any, supervision, 3. overburdened with extra-curricular activities, and 4. teaching a variety of courses. He may not have decided, as yet, which of the subject matter areas he prefers and he will certainly not join all of the following: the National Science Teachers Association, the National Council of Teachers of English, the National Council for the Social Studies, *and* the National Council of Teachers of Mathematics. If he should join even one, which would it be?

This teacher, after he has served his apprenticeship, will probably move on to a larger school where he will be in a position to exert considerable influence and leadership in the teaching of mathematics.

In other words here is a group of neophytes who need our help *now*. Given proper assistance and encouragement, some will become tomorrow's leaders.

### *How many such schools and teachers are there?*

According to the 1946 report from the U. S. Office of Education, there are about 600,000 pupils and 40,000 teachers in these small high schools (4-year and junior high only). It is perhaps more meaningful to state that such schools include about 13% of all high school youth and about 20% of all high school teachers in the United States.

A more dramatic, but perhaps less pertinent, view of the situation can be gained by an examination of the 1950 census figures:

| *Population* | *No. of Cities in U. S.* | *Cumulative* |
|---|---|---|
| 100,000 (plus) | 106 | 106 |
| 50,000–100,000 | 130 | 236 |
| 25,000– 50,000 | 273 | 509 |
| 10,000– 25,000 | 918 | 1,427 |
| 5,000– 10,000 | 1,442 | 2,869 |
| 2,500– 5,000 | 1,882 | 4,691 |
| 1,000– 2,500 | 4,857 | 9,548 |
| 0– 1,000 | 112,984 | 122,532 |

It seems reasonable to assume that the small schools are, by and large, found in communities whose population is less than 1,000—certainly in those of less than 5,000. As the table indicates there are a sizable number of such communities

Here, then, is a potential source of leadership and membership which we may be able to secure *if* we can make the National Council serve a real purpose to the teachers in the small schools. Are we reaching these teachers? I don't know how many are members of the Council—perhaps Mr. Ahrendt or Dr. Hildebrandt can tell us. I do know, however, that relatively few are attending our meetings.

I believe the largest attendance we have ever had at a convention was registered at the annual meeting in Chicago in 1950. According to Mr. Schreiber's report, a record of 1322 was established at that time. In terms of the addresses listed, the distribution according to population was as follows:

| *Population of Community Represented* | *Number in Attendance* |
|---|---|
| 100,000 (plus) | 565 |
| 50,000–100,000 | 179 |
| 25,000– 50,000 | 129 |
| 10,000– 25,000 | 241 |
| 5,000– 10,000 | 103 |
| 1,000– 5,000 | 89 |
| 0– 1,000 | 16 |
| | 1322 |

In other words, not more than 105 teachers, and perhaps as few as 16, came from communities which presumably contain small high schools.

*What can we do for the teacher in the small school?*

There are probably many activities in which the Council can, and does, engage and which could be aimed directly at the problems of the small school. I would like to mention three.

1. Give attention to these problems at our conventions.

   It is encouraging to note that Walter Carnahan is discussing "The Special Problems of the Small High School" at the summer meeting of the Council at the Phillips Exeter Academy. I see no reason why such topics could not be given a place on succeeding programs.
2. Publications
   a. A yearbook might be devoted to some of the problems of the small school. According to the April issue of THE MATHEMATICS TEACHER, "The Mathematics Curriculum of the Small High School" is one of twenty-three topics suggested to the current yearbook committee.
   b. There may be sufficient interest and available material for the inauguration of a new department in THE MATHEMATICS TEACHER.
   c. Perhaps the most promising publication possibility is to develop short, cheap, and easy to use materials similar to the "How to Do It" series published by the National Council for the Social Studies. This should, I believe be a part of the work of Henry Syer's "Pamphlet Committee."
3. The third suggestion is that we might contact the small schools through our Executive Secretary or an appropriate committee in order to learn first-hand what problems confront the teachers.

---

## ATTENDANCE RECORD OF THE THIRTIETH ANNUAL MEETING

### Hotel Fort Des Moines, Des Moines, Iowa
### April 16-19, 1952

| Name of State | No. | Name of State | No. | Name of State | No. |
|---|---|---|---|---|---|
| Alabama | 5 | Kansas | 13 | Ohio | 25 |
| Arizona | 1 | Louisiana | 3 | Oklahoma | 3 |
| California | 1 | Maryland | 3 | Oregon | 1 |
| Colorado | 8 | Massachusetts | 6 | Pennsylvania | 5 |
| Delaware | 1 | Michigan | 18 | Texas | 3 |
| District of Columbia | 11 | Minnesota | 40 | Utah | 1 |
| Florida | 5 | Missouri | 9 | Wisconsin | 17 |
| Georgia | 2 | Nebraska | 12 | Canada | 2 |
| Illinois | 92 | New Jersey | 5 | | |
| Indiana | 10 | New York | 15 | Total | 943 |
| Iowa | 625 | North Dakota | 1 | | |

# Report of the Third Delegate Assembly

MARY C. ROGERS, *Chairman*
*Committee on Affiliated Groups*
*Roosevelt Junior High School, Westfield, New Jersey*

THE THIRD Delegate Assembly of The National Council of Teachers of Mathematics was held at Des Moines, Iowa, Thursday and Friday, April 17 and 18, 1952. The agenda for the Assembly was prepared from suggestions of officers of Affiliated Groups and other members of the National Council.

This proposed agenda was published in the January 1952 issue of THE MATHEMATICS TEACHER. Members of the National Council were invited "to comment on these plans and to make proposals for their development so that The Third Delegate Assembly might be an effective means of providing more active participation of members everywhere in the growth and progress of the National Council of Teachers of Mathematics."[1]

Topics proposed for discussion concerned:

1. Inter-relationship of the National Council and the Affiliated Groups.
2. Special Activities initiated through the Affiliated Groups program.
3. The Publications of the National Council.
4. Improved means of determining problems of mutual interest to The National Council and to the Affiliated Groups.

In addition to the consideration of these topics, the program of the Assembly included: a welcome to the delegates by H. W. Charlesworth, President of the Council; the awarding of certificates of affiliation to newly affiliated groups; and the introduction of the delegates by John R. Mayor, Chairman of Affiliated Groups; and a report from the Washington office by M. H. Ahrendt, Executive Secretary.

### INTER-RELATIONSHIPS

The first topic discussed by the Delegates concerned the "General Nature of Programs at the National Council Meetings." The discussion indicated that the group favored the following:

"1. Laboratory periods, with the suggestion that the provisions made for the different levels be continued.
2. Continuity groups, some expressed an opinion that this type of grouping prevented participation in other phases of the program.
3. Discussion groups with interchange of ideas.
4. General sections arranged so there will be no conflicts with other meetings.
5. More programs which include practical help in teaching techniques and devices for teaching.
6. A workable plan for reporting discussion groups."[2]

"To the question on the 'Section sponsored by the Committee on Affiliated Groups,' the following motion was made by Guinevere D. White and seconded by Lenore John: that we continue a section sponsored by the Committee on Affiliated Groups with a program of a general nature. The motion was carried. A suggestion was made that the program avoid conflicts with other meetings, and that all members, rather than only those representing affiliated groups, be encouraged to attend."[3]

The Delegates favored "Sections sponsored by groups in the geographical area of the meeting" as a stimulant to interest and to wider program participation. They recommended a continuation of present practices in regard to the number of meetings held each year by the National Council.

In reply to the question "What obligations should an Affiliated Group which is host organization for a National Council meeting be asked to assume?" the following suggestions were offered by Agnes Herbert:

"1. Invitations must be received from the local group desiring the meeting.
2. Acceptance of invitation is limited by various factors such as facilities, location, amount of work entailed, and similar problems.
3. Mechanics of the program is the responsibility of the local organization and requires a large group of interested workers.
4. Invitations are always welcomed by the Council. The Council is anxious to receive invitations for the summer of 1953 and Christmas 1953."[4]

### SPECIAL ACTIVITIES

Considerable interest was shown in The Traveling Exhibit. It was suggested that the Exhibit Chairman at each Council meeting be responsible for the collection of material for this Traveling Exhibit. A motion was made and passed that we make use of any exhibit at the Council meetings if the owner of such exhibit

[1] John R. Mayor, "The Third Delegate Assembly" THE MATHEMATICS TEACHER, XLV (January 1952) 64.

[2] From the Minutes of the Third Delegate Assembly prepared by Alice M. Hach and Catherine A. V. Lyons, Secretaries of the Third Delegate Assembly.

[3] *Ibid.*

[4] *Ibid.*

is willing to lend it temporarily or to give it to the Council.

Difficulties arising from packing and the expense of mailing an exhibit were discussed. Slides and filmstrips were suggested as a convenient way to display material at a reasonable expense. The Chairman urged that teachers write suggestions to Miss Madeline Messner, 400 Chandler Avenue, Roselle, New Jersey.

"The Chairman presented a report, prepared by Mary C. Rogers, on the Speaker's Bureau, one of the activities sponsored by the Affiliated Groups. The purpose of this bureau is to provide a list of recommended speakers. Before trying out the Speakers' Bureau service nationally, a regional Speakers' Bureau is being set up experimentally in the New York area under the chairmanship of Mary Rogers. A year from now it should be possible to evaluate this experiment. A consensus showed that the group favored the Speakers' Bureau."[5]

### Publications of the National Council

A very interesting and complete report on The Mathematics Teacher was given by the Editor, E. H. C. Hildebrandt. The statistical data of this report appears at the end of this article. In addition to this factual information, certain points of mutual interest to editor and reader alike were stressed. There were:

"1. The Mathematics Teacher has a professional responsibility to reach every mathematics teacher.
2. There has been special effort to have delivery of the magazine to each member during the first week of the month. Issues for 1951–52 have followed this plan.
3. Members were urged to notify the Editor of people who would be willing to review books, evaluate articles, and perform other similar cooperative services for The Teacher.
4. There is a need for more publication space, more funds, more advertising, more good articles, more reviewers, and increased membership."[6]

"The report on the smaller publications as prepared by Henry W. Syer was given by the Chairman, John R. Mayor. The purpose of the Committee on Publications of Current Interest is to collect and prepare materials which are to be published in small pamphlets to sell at 5¢ to 50¢. Items on "How to Do," "How to Teach," test items, bibliographies, lists of free and inexpensive materials, ideas on clubs, and other items of pertinent value to teachers are among those being considered. Any suggestions are to be sent to Henry W. Syer, School of Education, Boston University, 332 Bay State Road, Boston 15, Massachusetts."[7]

A discussion as to whether this material should be furnished free, resulted in the following recommendation:

**A small fee should be charged by the Council for reprints and small publications.**

Veryl Schult, Chairman of the Yearbook Planning Committee for 1951–52 explained plans and progress of the Yearbook Committee. Hereafter the yearbooks will be known as *Mathematics Education Monographs Published by the National Council of Teachers of Mathematics.* A Planning Committee, consisting of F. Lynwood Wren, Chairman, Daniel Snader, and F. G. Lankford, will survey the needs of teachers and determine the type of monograph most useful and most generally requested. As soon as monograph subjects are chosen and approved by the Board of Directors of the Council, chairmen will be named to direct the writing and editing of the books. Three subjects were approved at the Pittsburgh meeting; work has been begun on the first two.

1. *The Learning of Mathematics—Theory Practice.* Chairman, Howard F. Fehr. This monograph is ready for editing
2. *Emerging Practices in the Teaching of Mathematics.* Chairman, John R. Clark.
3. *The Reorganization of Mathematics. Its Place in Education.*

Other topics submitted for consideration were: "Enriching the Teaching of Algebra and Geometry," "Second Track Mathematics," "Elementary Arithmetic," "Critical Thinking," "Evaluation," "Teacher Training and Preparation."

Delegates were urged to notify Dr. Clark of unusual work being done in mathematics—practices new, but proven to be educationally sound. They were also asked to write Dr. Wren giving ideas for monographs.

### Co-operative Planning

The success of any organization is directly commensurate with the interest and support of its members. The more people who can be brought into the deliberations and activities of an organization, the stronger it becomes. The National Council has increasingly encouraged co-operative planning as well as wide-spread participation by its members in all programs and services of the Council. Through this "working together" it has become one of the largest and most effective educational groups in the country.

In keeping with this philosophy and practice, the Chairman of Affiliated Groups requested that the Assembly appoint a committee from its own personnel to assist with details of policy and planning, and to aid in determining agenda for future sessions of the Delegate Assembly. A motion was made and carried that this appointment be made by the Chairman. The following committee was named: Mary Lee Foster, Daniel Lloyd, and W. V. Unruh. This committee met between sessions of the Assembly to determine the order of business for the second session and met again at the close of the second session to begin plans for the Fourth Delegate Assembly.

[5] *Ibid.*
[6] *Ibid.*
[7] *Ibid.*

### Funds

The Treasurer's report showed in summary:

| | | |
|---|---|---|
| *Balance April 1, 1951* | | $ 7.09 |
| *Cash Receipts* | | |
| Receipts from dues paid by Affiliated Groups | | 168.10 |
| Total Receipts | | 175.19 |
| *Disbursements* | | |
| *Newsletters* and postage | | 104.34 |
| Traveling Exhibit | | 10.00 |
| Badges for Delegates | | 19.91 |
| Bank Service charge | | .59 |
| Total | | $134.84 |
| *Balance April 15, 1952* | | 40.35 |

Discussion of the tentative budget for 1952–53 resulted in the following approximate allocation of funds:

| | | |
|---|---|---|
| Estimated Receipts from dues | | $200.00 |
| Expenditures | | |
| *Newsletters* and postage | $100.00 | |
| Traveling Exhibit | 50.00 | |
| Speakers' Bureau | 50.00 | 200.00 |

### New Groups

Certificates of Affiliation were given to the following eight groups affiliated since April 1, 1951:

Ohio Council of Teachers of Mathematics
Houston Council of Teachers of Mathematics
Mathematics Section of the Prince George's County Teachers Association, Maryland
Washington, D. C. Mathematics Teachers, Division 2
Southern New England Mathematics Association
Mathematics Section of the New Mexico Education Association
Beaumont Mathematics Association
Department of Mathematics of the North Carolina Education Association

### Official Delegate List

John R. Mayor served as chairman of the Third Delegate Assembly. Alice M. Hach and Catherine A. V. Lyons were secretaries of the Assembly. Special reports were presented by M. H. Ahrendt, Executive Secretary, Washington, D. C.; Agnes Herbert, Baltimore, Maryland; Dr. E. H. C. Hildebrandt, Editor of The Mathematics Teacher; Veryl Schult, Chairman of the Yearbook Planning Committee. Reports were submitted by Dr. Mayor for Henry W. Syer, Chairman of the Committee on Publications of Current Interest, and Mary C. Rogers, Chairman of the Regional Speakers' Bureau.

The official delegates were:

Mathematics Department, Alabama Education Association
Lucille Faulkner, Gadsen, Alabama
Arkansas Council of Teachers of Mathematics
Mary Lee Foster, Arkadelphia, Arkansas
California Mathematics Council
Dale Carpenter, Los Angeles, California
The Colorado Council of Teachers of Mathematics
Warren Geyer, Denver, Colorado
Florida Council of Teachers of Mathematics
W. A. Gager, Gainesville, Florida
Dade County Council of Teachers of Mathematics
Charlotte Carlton, Miami, Florida
The Pinellas County Council of Teachers of Mathematics
Mary L. Cantwell, St. Petersburg, Florida
Georgia Mathematics Council
Isabel H. Kinnett, Macon, Georgia
Illinois Council of Teachers of Mathematics
Helen A. Schneider, La Grange, Illinois
Chicago Elementary Teachers' Mathematics Club
Joseph J. Urbancek, Chicago, Illinois
Men's Mathematics Club of Chicago and Vicinity
Henry Swain, Winnetka, Illinois
Women's Mathematics Club of Chicago and Vicinity
Lenore John, Chicago, Illinois
Indiana Council of Teachers of Mathematics
Albert R. Mahin, Indianapolis, Indiana
Iowa Association of Mathematics Teachers
Herbert F. Tschopp, Ames, Iowa
Kansas Association of Teachers of Mathematics
W. V. Unruh, Merriam, Kansas
Wichita Mathematics Association
Anton S. Richert, Wichita, Kansas
Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics
Houston T. Karnes, Baton Rouge, Louisiana
Mathematics Section of the Maryland State Teachers Association
Agnes Herbert, Baltimore, Maryland
Mathematics Section of the Prince George's County Teachers Association
Margaret R. Myerly, Laurel, Maryland
Detroit Mathematics Club
Florence Jacque, Detroit, Michigan
Minnesota Council of Teachers of Mathematics
Edith M. Woolsey, Minneapolis, Minnesota
National Council of Teachers of Mathematics, Nebraska Section
Theodora Nelson, Kearney, Nebraska
The Association of Teachers of Mathematics in New England
Henry W. Syer, Boston, Massachusetts
Association of Mathematics Teachers of New Jersey
May J. Kelly, Atlantic City, New Jersey
Association of Teachers of Mathematics of New York City
Barnett Rich, New York, New York
Nassau County Mathematics Teachers Association
Alfred T. Anderson, Floral Park, New York
Ohio Council of Teachers of Mathematics
Mildred Keiffer, Cincinnati, Ohio
Oklahoma Council of Teachers of Mathematics
Velma Felkner, Norman, Oklahoma
Tulsa Mathematics Council
L. W. Lavengood, Tulsa, Oklahoma

The Mathematics Council of Oklahoma City Teachers
James Zant, Stillwater, Oklahoma
Ontario Association of Teachers of Mathematics and Physics
R. E. K. Rourke, Newmarket, Ontario
Pennsylvania Council of Teachers of Mathematics
Catherine A. V. Lyons, Pittsburgh, Pennsylvania
Western Pennsylvania Mathematics Teachers Association
Mabel Love Baker, Verona, Pennsylvania
Mathematics Section, East Tennessee Education Association
Kenneth E. Brown, Knoxville, Tennessee and Washington, D. C.
Houston Council of Teachers of Mathematics
H. Lel Red, Houston, Texas
Mathematics Section, Texas State Teachers Association
Ida May Bernhard, San Marcos, Texas
Greater Dallas Mathematics Association
Carrie Denson, Dallas, Texas
Richmond Branch of the National Council of Teachers of Mathematics
Allene Archer, Richmond, Virginia
Benjamin Banneker Mathematics Club, Washington, D. C.
Guinevere D. White, Washington, D. C.
Washington, D. C. Mathematics Club, Division 2
Daniel B. Lloyd, Washington, D. C.
West Virginia Council of Mathematics Teachers
J. C. Leake, Huntington, West Virginia
Wisconsin Mathematics Council
Dorothy Sward, Beloit, Wisconsin

STATISTICAL REPORT OF THE MATHEMATICS TEACHER FOR ISSUES OCTOBER 1951–MAY 1952 INCLUSIVE

| | | Pages |
|---|---|---|
| 1. Number of issues | 8 | |
| Pages per issue | 80 | |
| Total number of pages | | 640 |
| 2. Use of Space | | |
| a. Articles—42 (46 authors from 16 states, D. C., 3 foreign. Average length—5 pages) | | 206 |
| b. National Council Activities | | 66 |
| c. Departments: | | |
| Aids of Teaching | 51 | |
| Applications | 28 | |
| Books Section | 30 | |
| Devices for a Mathematics Laboratory | 35 | |
| Mathematical Miscellanea | 41 | |
| Mathematical Recreations | 22 | |
| Notes on the History of Mathematics | 10 | |
| References for Mathematics Teachers | 27 | |
| Research in Mathematics Education | 14 | |
| What is Going on In Your School? | 33 | 291 |
| d. Signal Corps Posters | | 16 |
| e. Advertising | | 39 |
| f. Index | | 10 |
| g. Page fillers; News Notes, etc. | | 12 |
| 3. a. Classification of Levels of Interest of Published Articles | | |
| Elementary | 6 | |
| Junior High School | 10 | |
| Senior High School | 21 | |
| College | 8 | |
| Mathematics | 4 | |
| General Interest | 12 | |

b. Number of Items Published in the Departments According to Levels of Teaching Interest

| | *Elem.* | *Jr. H.S.* | *Sr. H.S.* | *Col.* | *Tchr. Tr.* |
|---|---|---|---|---|---|
| Aids | 47 | 88 | 101 | 21 | 4 |
| Applications | 2 | 15 | 25 | 6 | |
| Books | 4 | 6 | 15 | 37 | 8 |
| Devices | 3 | 10 | 19 | 4 | |
| History | 3 | 5 | 5 | 4 | |
| Miscellanea | 0 | 5 | 26 | 17 | |
| Recreations | 0 | 3 | 9 | 4 | |
| References | 1 | 7 | 8 | 6 | 5 |
| Research | 3 | 4 | 4 | 1 | 8 |
| What Is Going On | 2 | 4 | 19 | 6 | 2 |

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**Reprints of "Mathematical Preparation for College,"** which appeared in the May 2, 1952 issue of THE MATHEMATICS TEACHER, are now available. This article, addressed to high school students, shows the mathematics needed in preparation for various vocations as revealed by recent studies. Reprints are available with or without covers. Price with covers, 20¢ each; in quantities of 10 or more, 15¢ each. Without covers, 15¢ each; in quantities of 10 or more, 10¢ each. Send your order with remittance to the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C.

# Announcing the Twenty-First Yearbook of the National Council of Teachers of Mathematics

## The Learning of Mathematics—Its Theory and Practice

How does the human brain and nervous system acquire its store of mathematical knowledge? How does the human organism use this store of knowledge once it has acquired it? How can teachers direct the behavioral growth of their students so that they acquire and use mathematical knowledge? These are the fundamental questions to which the answers can be of great aid in the improvement of instruction in mathematics. Although very little is known about the answers to how we learn, the little that is known should be studied by every teacher of mathematics on every level of instruction from kindergarten through college. This yearbook has been written to provide some of this information, and to indicate how it may be put into effect by the classroom teacher.

How we learn is described in part by physical processes, by psychological aspects of behavior, and by philosophical considerations. In all descriptions there are present the elments of maladjustment, insecurity, dissatisfaction, motivation, drive, set, emotional disturbance, diagnosis, problem realization, recall, associations, trial and error (approximation and correction), analysis, hypothesis, formation, solution, insight (illumination), goal attainment, knowledge structure, logical organization, energy relief, and satisfaction. This yearbook discusses these scientific aspects of learning with down-to-earth, concrete descriptions, and illustrations within the field of elementary, secondary, and collegiate mathematics, so that the classroom teacher who is a non-specialist in psychological theory can understand and use them.

The questions raised by this whole theory of learning are numerous. How is the organism motivated? How does it form its attitudes and habits? How are the senses directly related to learning mathematics? What is a concept, and how are concepts formed? To what extent does the proper use of language aid the learning of mathematics? Is learning achieved with the first intial success, or is practice (drill) a necessary requirement of learning? Do the procedure of learning in mathematics and the modes of thinking thus acquired transfer to later learning and to learning in other fields? What is meant by saying the pupil has a *problem*, and how does the organism learn to solve problems? Do learners differ in ability to achieve, and in ways of achieving, and if they do, how do we provide for the various rates of learning of the students within a given class? Must learning be planned, or is it a hit or miss affair? How can we use the answers to these questions to improve our daily instruction?

The yearbook has given the best answers to these questions that are available today. You will certainly want to read it, study it, and have it on your desk as a guide and reference book. The book will be sold by the Office of the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C. Look for the exact date of publication and price of the book in the next issue of The Mathematics Teacher.

The contributors are:

Howard F. Fehr, New York City. (Editor)
Irvin H. Brune, Cedar Falls, Iowa.
John R. Clark, Lahaska, Pennsylvania.
Irving A. Dodes, New York City.
Maurice L. Hartung, Chicago, Illinois.
Kenneth B. Henderson, Urbana, Illinois.
Robert E. Pingry, Urbana, Illinois.
Myron F. Rosskopf, New York City.
Rolland R. Smith, Springfield, Massachusetts.
Ben A. Sueltz, Cortland, New York.
Henry W. Syer, Boston, Massachusetts.
Henry Van Engen, Cedar Falls, Iowa.

# Membership Message

*By* MARY C. ROGERS
*Roosevelt Junior High School, Westfield, New Jersey*

ANOTHER school year is well on its way. Vacation days, with their many pleasant memories, have receded into the past—replaced by days and weeks of work and initial achievement. Plans for your new year are doubtless already being materialized and promise much in educational service through cooperative endeavor with those around you. National Council gives you its best wishes for the most successful and satisfying year in your professional experience.

It has been very pleasant meeting and talking with many of you at the various conventions and conferences during the past year. We have been impressed by your loyalty to the National Council and by your enthusiastic desire to give its programs and activities your best support.

Among other things, you have talked about membership. You have voiced an appreciation for the invaluable aid afforded you by THE MATHEMATICS TEACHER, and for the other services you are receiving as a member of the National Council. You have expressed a sincere wish that more teachers of mathematics would come to realize and benefit from such membership. You have shown interest in the membership news we have given you in the past, and have asked that the dissemination of this news be continued.

During the past two years we have told you much about *membership growth*. Our major emphasis in this message concerns an analysis by states of total membership. We are showing you on page 415 how membership in the various states is distributed as to *individual* membership vs. *institutional* memberships (memberships carried by schools, school libraries, etc.). To this information we are adding certain other analyses which we hope you will find interesting and helpful. The data from which this report is made were furnished us from the office of the Executive Secretary in Washington, D. C.

In seventeen states 80% or more of total membership is made up of memberships by individuals, there being relatively few institutional memberships. These states are:

| | | | | | |
|---|---|---|---|---|---|
| District of Columbia | 90% | Illinois | 86% | Maryland | 82% |
| West Virginia | 89% | Rhode Island | 86% | Massachusetts | 82% |
| Kansas | 87% | Indiana | 84% | Maine | 81% |
| Arkansas | 86% | Iowa | 84% | Louisiana | 80% |
| Delaware | 86% | Colorado | 83% | Florida | 80% |
| | | Montana | 83% | | |

### STATES SHOWING MOST GROWTH, MAY 1951-MAY 1952

| | | | |
|---|---|---|---|
| Iowa | California | New Mexico | Washington |
| Texas | Colorado | Connecticut | Georgia |
| Illinois | Oklahoma | Montana | Minnesota |

### STATES SHOWING GREATEST RELATIVE GROWTH, MAY 1951-MAY 1952

| | | | |
|---|---|---|---|
| Iowa | Colorado | Vermont | Washington |
| Montana | Nevada | Maine | South Dakota |
| New Mexico | Texas | North Dakota | Oklahoma |

### STATES LEADING IN MEMBERSHIP TOTALS

| | | | |
|---|---|---|---|
| Illinois | Texas | Indiana | Massachusetts |
| New York | California | New Jersey | Iowa |
| Pennsylvania | Ohio | Michigan | Wisconsin |

It has been suggested that we make further membership analysis on a pro rata basis. Not being able to learn the number of teachers of mathematics in the various states, we have determined the total population by states and then pro rated the membership returns for 1951–1952 by the 100,000 population. This study gives the following results:

| States | Membership Returns per 100,000 Population |
|---|---|
| Delaware | 12.5 |
| Iowa | 11.3 |
| Colorado | 11.0 |
| District of Columbia | 11.0 |
| New Mexico | 10.2 |
| Kansas | 10.1 |
| Illinois | 9.7 |
| Indiana | 9.5 |
| Florida | 9.3 |
| Nebraska | 8.8 |
| Minnesota | 8.5 |
| Oklahoma | 8.3 |

As has been our custom for the past three years, we shall again this year give special recognition to those schools with outstanding membership records—our 100% and "All but One" schools. We shall publish two listings of these Membership Honor Schools. The first will appear in the February 1953 issue of THE MATHEMATICS TEACHER. A second Honor listing is planned for the May 1953 TEACHER.

We need your assistance in locating these schools. We shall greatly appreciate your cooperation. Will you help us? Simply complete the accompanying form and mail it to

MARY C. ROGERS
307 Prospect Street
Westfield, New Jersey

If your reports are in our hands by November 15, 1952 we shall guarantee the appearance of qualifying schools in the February 1953 listing. May we look forward to hearing from a great many of you. Thank you very much.

MEMBERSHIP REPORT TO THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

School.....
School Address.....
Number of Teachers of Mathematics in Your School.....
Number of National Council Members.....
Report Submitted by.....
Your Address.....
Names of Individual Members.....

## Committee on Affiliated Groups

IN ORDER to more effectively serve the Affiliated Groups and to better co-ordinate their activities, the membership of the Committee on Affiliated Groups will be increased to four, representing geographical areas, and a chairman. Applications for affiliation and renewals of affiliation are to be sent to the regional representatives on the Committee.

The Committee on Affiliated Groups will send a statement of details of the new organization directly to each Affiliated Group, and reports from the Committee will appear regularly in THE MATHEMATICS TEACHER.

Mary C. Rogers, Roosevelt Junior High School, Westfield, New Jersey, has been appointed chairman of the Committee. Regional representatives are:

SOUTHEASTERN: W. A. Gager, University of Florida, Gainesville, Florida
(Alabama, District of Columbia, Florida, Georgia, Kentucky, Louisiana, Mississippi, Maryland, North Carolina, South Carolina, Tennessee, Virginia, West Virginia.)

SOUTHWESTERN: Ida May Bernhard, San Marcos High School, San Marcos, Texas
(Arizona, Arkansas, California, Colorado, Kansas, Missouri, Nevada, New Mexico, Oklahoma, Texas, Utah, Wyoming.)

NORTHWESTERN: Donovan Johnson, University of Minnesota High School, Minneapolis, Minnesota
(Idaho, Illinois, Indiana, Iowa, Michigan, Minnesota, Montana, Nebraska, North Dakota, Oregon, South Dakota, Washington, Wisconsin.)

NORTHEASTERN: Jackson Adkins, Phillips Exeter Academy, Exeter, New Hampshire
(Connecticut, Delaware, Maine, Massachusetts, New Hampshire, New Jersey, New York, Ohio, Pennsylvania, Rhode Island, Vermont and Ontario.)

# Membership Campaign

All members of the National Council of Teachers of Mathematics will be interested to know of and to participate in the membership campaign that has been planned for the coming school year. There are several reasons why a membership campaign is desirable at this time. 1. When an organization stops growing it is in danger of losing some of its vitality and vigor. 2. The critical shortage of engineers, scientists, and other technical manpower makes it important that we do everything possible to strengthen the teaching of mathematics. 3. The income of the Council is hardly adequate to meet the expenses of our expanded program.

The membership campaign will be conducted through the state representatives and their assistants. The representatives will work under the direction of a new Committee on Membership, composed of the following persons: W. A. Gager, Henry Swain, Jackson B. Adkins, Kenneth Brown, Lucy Hall, H. Van Engen, Clifford Bell, Elizabeth Roudebush, and M. H. Ahrendt, chairman. The committee is preparing new publicity materials and other aids for use in the campaign.

The help of all members of the Council is needed to make the campaign a success. Sell membership in the Council to your colleagues who have not joined. Give your state representative all the help and encouragement you can. The present membership of the Council is about 8,800. Our immediate goal is 10,000. Help us reach that goal! We cannot fail if *every* member will help!

M. H. AHRENDT, *Executive Secretary*

## MEMBERSHIP IN THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

**May 1, 1952**

| *State* | *Individual* | *Institutional* | *Total* |
|---|---|---|---|
| Alabama | 81 | 30 | 111 |
| Arizona | 25 | 12 | 37 |
| Arkansas | 67 | 11 | 78 |
| California | 225 | 159 | 384 |
| Colorado | 105 | 21 | 126 |
| Connecticut | 107 | 37 | 144 |
| Delaware | 30 | 5 | 35 |
| District of Columbia | 102 | 11 | 113 |
| Florida | 176 | 44 | 220 |
| Georgia | 86 | 33 | 119 |
| Idaho | 5 | 3 | 8 |
| Illinois | 643 | 104 | 747 |
| Indiana | 272 | 52 | 324 |
| Iowa | 214 | 42 | 256 |
| Kansas | 156 | 23 | 179 |
| Kentucky | 47 | 21 | 68 |
| Louisiana | 130 | 32 | 162 |
| Maine | 25 | 6 | 31 |
| Maryland | 138 | 31 | 169 |
| Massachusetts | 220 | 48 | 268 |
| Michigan | 198 | 81 | 279 |
| Minnesota | 162 | 53 | 215 |
| Mississippi | 55 | 20 | 75 |
| Missouri | 106 | 56 | 162 |
| Montana | 29 | 6 | 35 |
| Nebraska | 84 | 23 | 107 |
| Nevada | 2 | 5 | 7 |
| New Hampshire | 26 | 8 | 34 |
| New Jersey | 234 | 67 | 301 |
| New Mexico | 38 | 16 | 54 |
| New York | 506 | 169 | 675 |
| North Carolina | 116 | 47 | 163 |
| North Dakota | 13 | 9 | 22 |
| Ohio | 311 | 70 | 381 |
| Oklahoma | 131 | 40 | 171 |
| Oregon | 56 | 16 | 72 |
| Pennsylvania | 424 | 146 | 570 |
| Rhode Island | 30 | 5 | 35 |
| South Carolina | 52 | 30 | 82 |
| South Dakota | 20 | 9 | 29 |
| Tennessee | 99 | 39 | 138 |
| Texas | 341 | 125 | 466 |
| Utah | 24 | 12 | 36 |
| Vermont | 17 | 8 | 25 |
| Virginia | 169 | 46 | 215 |
| Washington | 46 | 40 | 86 |
| West Virginia | 79 | 10 | 89 |
| Wisconsin | 190 | 52 | 242 |
| Wyoming | 13 | 8 | 21 |
| Totals | 6425 | 1941 | 8366 |
| U. S. Possessions | 21 | 14 | 35 |
| Canada | 78 | 51 | 129 |
| Foreign | 86 | 117 | 203 |
| **Grand Totals** | 6610 | 2123 | 8733 |

# Mathematics and Manpower*

*By* Allen Orth
*General Motors Corporation, Detroit, Michigan*

Before accepting the invitation to discuss this subject, I debated with myself at some length as to my fitness to do an intelligent job. As an engineer by education, I do, or rather did, possess a certain familiarity with mathematics. I say "did" because in my present capacity I rarely come in touch with involved mathematics except on occasions like that of last month when I made out my income tax report.

But my present activities do bring me into rather frequent contacts with some of the nation's industrialists and as a result I have gained the impression that they are worried about a situation that should cause us all to do quite a bit of thinking. So, in this discussion it would be better if you looked upon me *not* as an engineer, *not* as an amateur mathematician, but rather as a catalyst.

Let us take time out for a minute to look at the world situation. To put it mildly, we are not loved by all the peoples of the world, in fact, two of the nations with the largest populations have taken an active dislike to us—a dislike that hangs over our heads like the sword of Damocles. These nations are equipped with trained armies numerically greatly superior to ours and these armies seem to be supplied with adequate weapons—guns, tanks and planes.

We are in the midst of preparations to make ourselves strong enough to resist aggression but we cannot hope to compete on the basis of sheer numbers. Our only hope is to develop and produce more and better weapons in a comparatively short time.

We must have trained scientists to unravel the mysteries of nuclear forces, the fundamentals of guided missiles and the chemistry of warfare. There must be engineers and technicians to design, test and produce the things that result from this research and see that they come out of the factories in quantities to make the difference between victory and defeat.

Please don't misunderstand my emphasis at this time on these military applications of science and engineering. Existing world conditions now, and for some years to come, necessitate this emphasis. But at the same time we should not forget that an atomic powered airplane or ship can carry passengers, a rocket can explore space, or that synthetic rubber tires can carry a tank of milk to the dairy just as they carry a tank of fuel to the vehicles up front. Scientists and engineers are the architects of victory in times of war and of prosperity in times of peace.

But now we come to the thing that is disturbing many leaders around the nation. If our country is to be strong and to prosper, we need a steady flow of new scientists and engineers to take care of our expanding defense and peace time requirements. You all have seen the advertisements in our papers across the country calling for mechanical, electrical, metallurgical, chemical, civil engineers and research scientists. Where are we going to get them? In last September's issue of *Scientific American* an analysis was made of the human resources of the United States which gives us a good summary of our situation.

I would like to give you a résumé of the findings of just two authorities whose articles appeared in this issue—M. H. Trytten, Director of the Office of Scientific Personnel of the National Research Council and Karl T. Compton, Chairman of the Corporation of the Massachusetts Institute of Technology.

* Presented at the 30th Annual Meeting of the National Council of Teachers of Mathematics, Des Moines, Iowa, April 18, 1952.

Dr. Trytten's observations deal with scientists. We have 175,000 of them in the United States. Half are employed by private industry, a fourth by government agencies and a fourth by the universities. Since 1940 the demand for scientists has increased by leaps and bounds. In 1940 industrial laboratories employed 36,000 research workers, in 1946, 54,300 and in 1950, 70,500.

A year ago 16 companies employing scientific personnel sent their scouts around to the colleges to estimate the potential scientific employees in the June graduating class. They came back with the sad story that they would obtain only about *36 per cent* of the new employees they needed. In 1951, 47,000 students majoring in science were graduated, this year the estimate is 36,000, in 1953 the new supply will be about 33,500 and for 1954 the forecast is 29,000. And these figures do not allow for the demands of the armed services which may cut them in half.

So, in the next few years we are faced with a constantly increasing demand to be filled with a constantly decreasing supply. Such a situation, as can easily be seen, can have serious implications and could greatly handicap our development programs for national leadership in either peace or war.

With respect to engineering manpower requirements and the potential supply, Dr. Compton finds the situation equally critical. The numbers of engineers in this country have grown from 25,000 in 1890 to 400,000 in 1950. It is our third largest profession. Private industry employs three-fourths of these, the government 90,000, and 10,000 are on the staffs of educational institutions. There is a good reason for this. A hundred years ago manpower and animal power produced 94 per cent of our goods and machines produced only 6 per cent. In 1960 the reverse will be true, machines will produce 96 per cent of our goods and men and animals only 4 per cent.

This change in our way of doing things is responsible for the increasing demand for engineers in all categories. They design the machines and the things the machines produce. Sixty years ago there was only one engineer needed for every 390 workers but in 1950 one was required for every 60.

So we need engineers—thousands of them. In fact, it is generally estimated we need a minimum of 30,000 new ones per year.

How many are we getting?

Last year—*and remember, these figures do not take into account the demands of the Armed Services*—last year 38,000 were graduated, 1952 will see about 26,000 get their degrees, 1953, the figure will be about 17,000 and Dean Hollister of Cornell, in an estimate for the Engineering Manpower Commission sets a figure of 12,400 for 1954! Right now it is estimated we have a back log of demand for over 60,000 engineers to fill existing jobs. *This in the face of the report that Russia is now graduating over 50,000 engineers per year*!

Why are we running into this critical situation now? There are a number of reasons for this paucity, but I'll give you two or three of the generally accepted ones. College enrollments have fallen off due to two factors, the low birth rate in the early thirties and the completion of the courses taken by the large numbers of GI's immediately following World War II. We had the unfortunate statement which was widely publicized in the spring of 1950 which said in effect, industry is using an increasing number of engineers and would perhaps employ 450,000 by 1960 *"but so many engineering students will be graduated in the next few years that many graduates will be unable to get an engineering job."* These factors plus the rapidly accelerating demand for engineers coupled with the superimposed needs of the Armed Services has led to the present critical situation.

There are not many things we can do to increase the supply of these engineers and

scientists in the next four years. The die is cast. However, there are some steps we can take with respect to those *now in college*. This was the subject of an Institute of Science Teachers held under the auspices of the Thomas Alva Edison Foundation last November in West Orange, New Jersey.

I'll itemize these suggestions as they were given remembering, of course, these apply to universities and colleges of engineering and science.

1. Take a hard look at engineering drop out rates. Estimates indicate that about 60% of each engineering class does not graduate.
2. Establish intimate relationships with secondary school administrators and science teachers in order to remove any barriers causing misunderstandings.
3. Establish opportunities and courses, basic to science and engineering, for local secondary and elementary school science teachers to increase their teaching effectiveness.
4. Survey enrolled liberal arts students to locate those who may have been directed away from science and engineering by the over-supply estimate of 1950.
5. Examine course offerings and determine the extent and degree of mathematical proficiency that is required for different career opportunities in engineering and science.
6. Establish educational programs for individuals who have dropped out of engineering and scientific schools, who though they may lack professional ability would make satisfactory technical assistants.
7. Encourage more young women to take up careers in engineering and science.
8. Emphasize the formal evaluations and encouragements of individuals based on their originality and not their "talent" to memorize the achievements of others.

These are some of the things that can help alleviate the present situation. In other words leave no stone unturned to make certain no college student now enrolled lacks an opportunity or encouragement to improve his technical talents.

But I am sure we are all interested in what happens in this respect beyond 1954. At that time it looks as though we shall have accumulated an even greater backlog of demand for scientists and engineers. The important thing then is to focus our attention on those now in high school or even junior high. In those students lies our world leadership.

At the previously mentioned Edison Institute meeting held last fall a great deal of emphasis was placed on steps to be taken in this area to improve the quantity and quality of future scientists and engineers. Because many of the conclusions arrived at impinge on the activities of you people listening to me tonight I believe I should briefly review them:

1. Work through state departments of education to increase the availability of correspondence courses in mathematics and science for able students in high schools where there are too few to warrant offering classroom courses.
2. Cooperate with state departments to undertake critical studies to discover the factors that cause high enrollment and continued interest in mathematics and science in some schools and the opposite in others.
3. Examine history courses to be sure there is adequate emphasis on the economic and social impact and significance of accomplishments of science and engineering.
4. Expand teacher and student visits to local industries to obtain personal observations of what engineering and scientific jobs require and accomplish.
5. Provide enthusiastic leadership and interest among the teaching staffs for initiating and maintaining science, engineering, mathematics, and other related forms of clubs and activities including such things as science and mathematics clubs, talent searches, fairs, contests, congresses and assemblies.
6. Recognize the real individual differences in interests, abilities, modes and rates of learning that characterize youth.
7. Encourage superintendents, principals and science teachers to actively and energetically seek mutual working relationships with industrial and business organizations of each community which might serve the school in relation to engineering and science training.
8. Conduct constant, thorough and exhaustive studies on the causes for dropouts in high school. Current statistics show more than fifty percent of those entering high school drop out before graduation.
9. Improve science courses and science training of teachers at the elementary school level in order to develop scientific interests of children when they have the

greatest urge to learn.
10. Stress the importance of manual skills.

I know many of these suggestions are not new to you and I also am sure most of you have been implementing some of these ideas in your daily classroom teaching. To those of you who are doing this we owe a vote of thanks for your constructive contribution to our nation's future.

From the viewpoint of the outsider I know the teaching of mathematics can often be a thankless job. The cards have been stacked against you for generations. I happen to have a son who is in his Junior year in engineering. I recently received a letter from him in which he joyfully proclaimed he had just passed differential equations and he was through with mathematics. This, in spite of the fact that since the day he started school I have taken great pains to impress upon him the value of manipulating numbers and have spent hours demonstrating to him how useful such knowledge can be to a future engineer. I am sure that during the past fourteen or fifteen years he has had at least a dozen excellent teachers who have attempted to instill into him and his classmates an appreciation of this science. Please don't misunderstand me. This young fellow does not dislike mathematics. In fact I think he rather likes to juggle equations. When the time comes I believe he will use it instinctively.

Recently I was visiting a friend over in our research laboratories and I asked him a question, "Bob, how deeply do you fellows over here go into mathematics? What kind of stuff do you use in your daily work?"

Bob, who belongs to a mathematical society and is a department head, said, "From time to time I have tried to find the answer to that one. In fact, just the other day some of the boys were working on a tough one and just out of curiosity I asked what was their mathematical approach. One of them looked up from his desk and said, 'We haven't gotten around to any mathematical analysis yet, we are just trying to get this data straightened out.' He was absolutely sincere about this, yet while he was talking to me he was holding a slide rule in one hand. You see he used mathematics so instinctively he was unaware of it—just like reaching for the telephone when it rings."

My friend Bob, who is an old hand at automotive research, has reached some conclusions that may be of interest to you whose job it is to create an understanding of mathematical processes and their uses. This is the way he puts it. Mathematics, like any other art or craft that we use in our work, requires both tolls and materials. The tools are the symbols and operations of mathematics and the raw material is our knowledge of the laws of nature and of factors acting in the processes we want to investigate. In any profession, tools are constantly being improved. Even with the best tools, however, we will get a poor product if we lack proper materials.

In the early days of the automotive industry, engineers knew very little about the processes that take place in engines and chassis, so mathematics was almost useless for lack of raw material. Knowledge had to be gathered by experimentation. Gradually, as the stock of information increased, analysis has become more and more important, and today, a good research engineer must be at least a fair mathematician in order to make use of knowledge that others have accumulated. When we speak of mathematics we not only mean calculus, differential equations and all the other branches which are often called higher mathematics, but also the more modest branches of arithmetic, algebra and geometry. The last three are used every day but the more difficult methods find their application too, and their ratio of success to failure is probably about the same as that of purely experimental methods.

Let us take, as illustration, a case example of the analytical approach. The problem was to get the facts on lubrication and

load capacity of plain bearings under the the influence of a variable load as for instance, the load capacity of the bearings on the reciprocating parts of an automobile engine. There was text book theory on bearings but it held good only for a steady load and the experimental data available on variable loads were not complete enough for a final conclusion. So a new, more general theory was developed which showed how the lubricant should act and how the load capacity should change when the load and the bearing both rotated at various speeds. Enough was known of the process to devise a series of tests which covered all possible conditions and when these tests were run they confirmed the conclusions of the new theory.

In cases of this kind, detailed numerical analysis performs a job that cannot be done by experimental work or any other method. It brings order and system to an accumulation of unorganized data.

However, this work is limited by the inventiveness and ingenuity of the mathematician, and even more by the speed at which he can work. The gifted man will make the right guess early in the game and then prove it is correct. But even the dullest investigator might finally hit on the right clue if he keeps trying all the possible assumptions. This is where the rapid computing machines come into the picture. The saving in computing time they promise will let us carry out many trial analyses, even of complicated processes, until we find one that checks the facts.

At this point I would like to recall to you a couple of formulas because they not only have a tremendous impact on our way of life today but will exert a greater influence on our future.

The first of these was the result of a deduction by Albert Einstein in 1905 when he concluded that the amount of energy, $E$, equivalent to a mass, $M$, was given by the equation

$$E = MC^2$$

where $C$ is the velocity of light. If we state this in actual numbers the formula becomes more startling. For instance, if we could convert one kilogram of matter, or 2.2 pounds, entirely into energy we could theoretically obtain 25 billion kilowatt hours of energy as compared to the usual 8.5 kilowatt hours of heat energy produced by burning a kilogram of coal.

For the next 25 years, scientists, philosophers and engineers speculated on the possibilities but without supporting experimental evidence. But around 1930 some tangible experimental results began to bear out the theory. This evidence accumulated by leaps and bounds in the next ten years culminating in the first atomic bomb explosion on July 16, 1945 at Alamogordo Air Base in New Mexico. Thus the Atomic Age was ushered in, forty years after Einstein wrote down a formula.

As impressive as this development is, the average American today is very little influenced by it in his day-to-day routine except possibly he might come into contact with some of its by-products in a medical way. But he does hear and read about such things as atomic-powered ships and airplanes and the hydrogen bomb. But by and large these seem to be for the future. Undoubtedly the formula $E = MC^2$ has opened the doors to an entirely new world but I am afraid quite a few years will pass before we find ourselves part and parcel of it.

There is also another formula that has changed our entire mode of living in the last half century but if you mentioned it to the man on the street he would probably look at you with a blank stare. Here it is

$$e = 1 - \left(\frac{1}{r}\right)^{k-1}$$

in which

$e$ = efficiency

$r$ = ratio of compression

$k$ = ratio of specific heats at constant pressure and volume

About seventy-five years ago Dr. Nicholas August Otto built a 4-cycle gasoline engine around this formula. And by 1892 his engine was being used in stationary and marine applications. Still later it found its way into horseless carriages. Taking a look at the formula it is obvious if we just increase "$r$," raise the compression ratio, or keep squeezing the mixture of air and fuel in the combustion chamber more and more, the engine will become more efficient, using less gasoline and getting more power.

But why didn't the engine builders take advantage of this? The answer was very simple as Charles F. Kettering pointed out to the Society of Automotive Engineers in 1919. When the compression ratio was increased above 4 or 4½ to 1, the engines knocked, lost power and overheated. It seemed that this automatically put a ceiling on the formula at these compression ratios.

Research men, like Mr. Kettering, hate ceilings to progress so they set out to get to the bottom of the thing called "knock." They found the answer in the fuel used and the way it burned in the combustion chamber. The next job was even more difficult —to uncover, if they could, something they could add to the gasoline to change its character and eliminate the power robbing knock.

Then began a battle of cut and try. Thousands of compounds were tried over a period of at least five years, until the research men came up with the additive-tetra-ethyl lead. There were still more obstacles to be overcome before Ethyl gas began to get into the tanks of the motoring public. Then the engineers could really begin to do things with the compression ratios of the engines and make other design changes so that the power plants would shrink in size but deliver more power and economy. Thus the ceiling on the efficiency formula was lifted and motorists found they could go farther and faster on the same tank of gas.

Since the advent of these knock-resisting fuels, a new term has been added to our motoring language—"octane number" —which, as we all know, is simply a measurement of a fuel's ability to resist knock. In 1919 the anti-knock quality of the fuel was about 50 using a 4 or 4½ to 1 compression ratio. Today the octane number is around 90 and the compression ratios average about 7½ to 1.

"But," you may ask, "What is so phenomenal about all this? I don't see how this affects me, much less our nation."

I believe I can answer that one with a couple of figures. In these days when some people talk about our dwindling petroleum reserves I would like to point out that engineers, by taking advantage of the efficiency formula and making some mechanical changes in engines have saved us a third of our petroleum resources over the past 30 years and are saving the motoring public about a billion dollars a year! In the last twenty years the horsepower of the average automobile engine has doubled and at the same time our engines give more miles per gallon.

You do not want to under-estimate the power resulting from the application of that efficiency formula. For instance, by way of comparison, during 1950 the central power stations, that is electric light and power utilities, were capable of generating 80 to 90 million horsepower. In the same year in the new cars, trucks and buses produced by General Motors alone were power plants capable of generating a total of 350 million horsepower, almost four times as much as the combined capacity of all central power stations in the country.

If we consider that we have 53 million cars, trucks and buses running around the country we find under their hoods engines aggregating some 4¼ *billion* horsepower! So we owe billions of horsepower to that little formula.

These two formulas are just examples of hundreds that are constantly being

used in the day-by-day research, development and production which have made us the leading nation in the world. To implement these mathematical theories we must have the painstaking, patient and laborious efforts of thousands of scientists, engineers and technicians. These people, as I have pointed out, use nearly every conceivable type of mathematics in their daily work to unravel the mysteries of the unknown. They use this science as a tool to dig out new facts and apply old ones.

We need the theoretical type of mathematician who by logical deduction and a highly trained skill at manipulating involved mathematical processes can deduce new physical laws. We need the more practical type, who though not possessing the great skill of the theoretical mathematicians, yet can use less advanced mathematical processes to translate experimental data into a concrete application. In our highly technical world of today there exists no niche into which a person equipped with practically any degree of mathematical proficiency cannot be fitted.

Yet, as I mentioned earlier, there exists a definite reluctance on the part of many young people to become better acquainted with this science. You all know the usual reasons given for this antipathy such as mental laziness, lack of native ability and an ignorance of the science's ultimate usefulness. Even though a young person may have the ability and desire to increase his knowledge in this field he may not have the means nor do the local schools always offer an opportunity for him to further pursue his studies.

We must recognize these obstacles and take every means at hand to remove them. They are present in nearly every classroom where mathematics is taught today. Tremendous strides are being made in this direction through improved texts and methods of instruction, the mathematics laboratory and increasing educational facilities.

As I have tried to point out there has never before been a time in our nation's history when so many scientists, engineers and technicians are needed to maintain and increase the rate of our technical progress; and, unfortunately, there has never before been a time when the supply is going to fall so short of the demand.

It is not a case of trying to make every promising young person into an Einstein or an Oppenheimer but rather to see that no alert young mind lacks guidance and an opportunity to fulfill his or her capabilities. We in industry can and will be glad to show where the opportunities lie for these young people. In my own organization we are planning to invite school faculties and students to visit our plants and point out and discuss the many opportunities that exist for those with a technical training. Also we are planning a series of advertisements to be run in certain educational magazines correlating the sciences, including mathematics, with the design and construction of automobiles.

Ladies and gentlemen, this is a tremendous challenge to you. In your work you are doing a great deal more than inculcating mathematical principles. You are molding the future of our nation. We, in industry, clearly recognize your responsibilities and watch your efforts with great interest. We congratulate you on a job well done—a job that has been a major contribution to our world leadership. Working together, I am sure we can build a future such as the world has never seen.

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**Student Membership**

A SPECIAL MEMBERSHIP FEE FOR STUDENTS has been authorized by the Board of Directors of the National Council of Teachers of Mathematics. Any student who has never taught may become a member at the special rate of $1.50 for the first year. Applications must be submitted by a staff member of the school.

# A Report Card for Marking Achievement in Mathematics

*By* DONOVAN A. JOHNSON
*University of Minnesota, Minneapolis, Minnesota*

THE TASK of assigning marks to students is a time consuming, reoccurring task of every teacher. Although it may be a tedious task, it is also one which affords the teacher an opportunity to guide, evaluate, and motivate the students and to establish contact with parents, employers and educational institutions. Despite these important functions of marks teachers are not satisfied with the adequacy or accuracy of our marking systems. It is the purpose of this paper to make specific suggestions for the improvement of marks in mathematics.

It would seem essential that we first arrive at an agreement as to the purposes of marks. The following appear to be accepted functions of our present marking system:

1. Marks provide the student, parents, school authorities, and employers information as to the educational attainments of the student.
2. Marks provide one basis for the guidance of the student so that he is able to recognize and attain the maximum of his potentialities.
3. Marks provide a basis for advancement in our educational system since marks are used for transfer, classification, and promotion. Colleges and employers find marks worthy of consideration in the selection of candidates.
4. Marks provide a certain form of motivation that is a real force in the life of the student.

If our marks are to attain these goals we must answer certain questions. Some of these are "What attainments should our students achieve?" "How do we measure this achievement?" "What does a mark mean?" The answer to this first question involves a decision as to the objectives of mathematics. If we accept specific objectives such as knowledge of facts, skills of computation, problem-solving ability and ability to apply knowledge we will need to report progress in each of these areas. The same will be true for general objectives such as interests, habits, appreciations, and citizenship. Thus, it will be meaningless to give one mark to show progress in all or several of these aspects of our teaching. The acceptance of these objectives will also mean that we will need tools that will measure the attainments of a student in each area. Thus, the mathematics teacher will need broader tests that will measure many objectives. She will also need to use devices other than tests. She will need to observe the student's behavior in a variety of situations; she will need to check his procedures in solving problems; she will need to evaluate his work habits; and she will need to interpret words and actions in terms of attitudes and appreciations.

After we know a student's response to test items, problems, or situations, the problem of evaluating the responses arises. Should the student's achievement be compared to national norms, school norms, class norms, or to his own ability? Certainly the student's ability must be taken into account. But the uncertainty of knowledge of the *real* ability of a student makes it necessary to bring in comparisons with other members of the group. Other factors such as background, home conditions, time available, etc., need to be known. Also the area in which measurement is being made is a factor. It might be entirely unsatisfactory to determine a rating of work habits, interests, or behavior

on the basis of average class performance. Another factor that must be considered is the growth of the student. How much progress has he made? A student who has made little progress may score the highest on an achievement examination while the student with the lowest grade may have made significant growth.

Consequently, the form on which marks are reported should meet the following criteria:

1. It should report the status of the student with regard to his attainment of specific objectives. These objectives should include general objectives such as social adjustment as well as specific objectives such as the understanding of facts and principles.
2. The basis of the rating which the student is given should be stated clearly and specifically on the report form so that the parent, employer, student, or educational institution can interpret them properly.
3. The report should indicate the achievement of the student compared with his apparent ability to achieve. This is the true role of a report in motivating students. Again, the basis for determining a student's apparent ability needs specific description.
4. The report must be understood by non-professional persons as well as by educational institutions if it is to be accepted by parents, employers, and colleges.
5. The report must be feasible in terms of a teacher's time, energy, understanding, and evaluation materials.
6. The report should indicate special talents or outstanding progress as well as deficiencies or weaknesses.
7. The report should make recommendations as to how the student can attain his potentialities by capitalizing on strengths or repairing areas of weakness.
8. The report should indicate the status of the pupil in respect to important non-mathematical areas such as character development, work habits, or citizenship.

The report of actual progress as well as of potentialities for growth can be both absolute and relative. A report in the absolute sense is a description in terms of such non-personal or non-group criteria as e.g. "can solve 20 linear equations accurately in 10 minutes," or "can perform all required constructions." Marks of this nature are usually based on the performance of the student on a specific test or series of tests. A report of relative performance is in terms of (1) a group, e.g. percentile rank, grade norm, or a mark on the "normal curve"; or (2) potential performance of the individual, e.g. "not working up to ability," or "achieves a highly satisfactory mark by working beyond normal requirements." When an individual is marked relative to group performance, the selection of the group for comparison must be done carefully. Grade or national norms or local distributions vary greatly in meaning. In determining the basis for an estimate of the relative performance of an individual, the teacher should take into account the student's record in previous courses, his scholastic aptitude test scores, his mathematics aptitude test scores, and a subjective evaluation of the student's potentialities as exhibited to the teacher.

A comprehensive report will record a student's progress in both absolute and relative terms. The report should then be labeled completely so that the meaning of the mark is clear. It is obvious that a mark must be as accurate and fair as it is possible to make it. This means that several measurements made at different times and related to the different objectives will have to be made. Accurate and complete records should be kept so that a parent or student can be informed of the student's status at any time. Teachers should discuss the marking system with their classes so that the students are informed of the basis of marking. The report form itself

UNIVERSITY HIGH SCHOOL

Mathematics Department
Progress Report

Name _____ Quarter F W S Year 19___
Subject: Mathematics I II III IV V VI

| ACHIEVEMENT RATINGS | Above Average | Average | Below Average | Basis of Rating |
|---|---|---|---|---|
| Knowledge of facts | | | | Achievement compared with the achievement of all members of the class as measured by tests, projects, class work, assignments, etc. |
| Skill in Computation<br>Accuracy<br>Speed | | | | |
| Analysis of problem situations | | | | |
| Understanding of Principles | | | | |
| Ability to apply knowledge | | | | |

| ACTIVITY RATINGS | Good | Acceptable | Poor | Basis of Rating |
|---|---|---|---|---|
| Achievement compared with apparent ability to achieve | | | | Standardized test scores, previous achievement, scholastic ability test scores. |
| Work habits | | | | Independent and extra work, promptness, completeness and accuracy |
| Classroom Cooperation | | | | Cooperation, courtesy, dependability, attention, attitudes, etc. |

Mark for Quarter _____ Mark for Year _____

Comments:

Background in mathematics: _____

Remedial instruction needed: _____

Improvement: _____

Special projects: _____

Recommendation for future training: _____

Other: _____

_____
Instructor

should be outlined by the teacher but revised if desirable as based upon discussions with parents and pupils. Whenever possible the teacher should have conferences with students and parents to discuss the needs and progress of individual students.

In mathematics we are always concerned with the assumptions that are used as a basis for action. In devising a report form the following assumptions may *not* be valid:

1. A report must be made in terms of marks, either numbers or letters. A rating sheet or descriptive comments may be more suitable.
2. A report must be made for all pupils at a designated time. It might be more desirable to report at the end of a unit or topic.
3. The same type of report must be made in all subjects. Objectives and evaluation instruments will vary from subject to subject to such an extent that each high school subject will need a separate form.
4. A report should be made only in terms of academic achievement. Other objectives are important and need evaluation and reporting.

That such a report form is feasible is illustrated by the form below. A report form similar to this has been in use at the University of Minnesota High School for several years. It was developed by the mathematics department in cooperation with the entire faculty, students, and parents. It has meant the development of new tests, the keeping of extensive records, and renewed attention on the objectives of education. However, teachers, students, and parents are agreed that it is not perfect but that it is far more satisfactory than the traditional report that gives only an over-all achievement mark.

"Notice how I soared from 69 to 71 in arithmetic?"

From "Laughing Matter" by Salo, reprinted by special permission of the Chicago Tribune-New York News Syndicate.

# The Relation of Mathematics to the Core Curriculum

*By* HENRY S. JANSEN
*Baltimore Polytechnic Institute, Baltimore, Maryland*

A large group of educators is calling for a complete socialization of the high school program by the use of a core curriculum. This curriculum would take the common experiences of all fields of subject matter and combine them into an "integrated program" in which the pupil would learn by using all knowledge indiscriminately. Most forward looking mathematics teachers agree that we must stop compartmentalizing our mathematics by subjects and years, and proceed to fuse algebra, geometry, trigonometry, and analytics. into a single complete development of the science of number and space. But the core curriculum goes one better and includes not only this, but a fusing of all subjects into one single complete development of knowledge that will function in the life of every individual. Professor Fehr[1] feels that mathematics teachers are afraid of this because they are afraid the good ship mathematics may be lost in the process. The educators who are proposing the core curriculum are intelligent people. Let us then be perfectly frank and seek to learn if socialization by the use of the core curriculum is a desirable program in the high school, and if it is, how it can be achieved.

First, is there any tie-up of the mathematical instruction in any given year, say the ninth year, with the material presented in the other subjects? Has any attempt been made to present that mathematics in the ninth year that can be used in solving problems in the history class, problems relating to government financing, town surveying, housing, etc.? Do mathematics teachers know what is taught in ninth-year general science, and do they attempt to teach the algebra that could be used to help the pupil in his study of science? Have the other departments made any attempt to cooperate with the mathematics department in working out a coordinated program?

Secondly, do we have enough provision in our courses for experimentation and the development of the scientific method? Does the mathematics department combine with the art department in the study of dynamic symmetry and its use in beautifying our surroundings? Does it combine with the social studies department in a project of collecting data and analyzing the data to determine how much it costs to rear a child to the age of 18? Do we provide a laboratory where the application of geometric principles to machines can be studied at first hand? And would such studies be worth more to the great majority of our pupils than the present exposition of algebra and geometry?

Thirdly, do we use the library books other than our texts, the newspapers and pamphlets to enrich and broaden the mathematical knowledge and to show its relation to other fields of knowledge?

Finally, many teachers of mathematics take further study in courses showing the uses and applications of mathematics in the other fields of knowledge, thereby enriching their own knowledge and understanding. Do they apply this to their teaching or do they return to their schools and teach the same old mathematics in the same old way because they feel they do not have time to use this material? Perhaps they hesitate to experiment with these "new-fangled ideas," or feel "there

[1] Howard Fehr, "Socializing Mathematics Instruction," THE MATHEMATICS TEACHER, XLI, (January 1948), 4–7.

is no money to buy simple mathematical instruments," or decide they cannot change their classroom, or conclude that there is nothing to be gained by taking a class out of doors. We must admit that the educators may have a strong point to their criticism of the present instruction in mathematics. We cannot admit, however, that they have the best answer for improvement in their core curriculum. Let us see why not.

The fundamental aim of mathematics instruction was exceptionally well stated in the *Reorganization of Mathematics in Secondary Education* report of 1923. Here is the statement.[2]

> The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space, which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects; and to develop those habits of thought and of action which will make those powers effective in the life of the individual.

Will our students have an insight into and control over their environment under our present instruction? Does their present mathematical instruction give them an appreciation of the progress of civilization in its various aspects? If not, would the core curriculum be any better? Other worthy objectives of mathematical instruction are given in the Fifteenth Yearbook of the National Council of Teachers of Mathematics. They are: to develop fundamental skills, knowledge, and concepts; to develop desirable attitudes; and to initiate and develop interests and appreciations in the humanities, arts, and sciences that make for cultural growth, and richness of living. Do you believe that our present mathematical instruction attains to any marked degree, any of these objectives, except perhaps that of fundamental skills? If not, would the core curriculum do a better job? It cannot be denied that mathematics is a large integral part of our modern culture. The humanitarians and educators know this, and what worries many of them is that they do not understand the real nature of mathematics because they have not studied it sufficiently. They are therefore unable to see its place in the total program of education, although they sense the need. Today there is hardly a field of knowledge that has not been impregnated with mathematical modes of exposition. Yet the high school teachers in these other fields of knowledge are for the most part ignorant of mathematics and mathematical modes, and hence avoid all mention of mathematics in the subjects they teach. Many of these teachers are afraid of mathematics. Until the teachers of English, social studies, science, art, music, etc., are required to study sufficient mathematics to understand its basic principles, and its important role in their own field and in modern culture, the socialization of mathematical knowledge by the use of a core curriculum must be carried out by the mathematics teachers in their own classes. The past training of teachers further justifies this procedure. In teachers colleges the barest minimum of mathematics instruction is required. In some cases this consists of a single course in arithmetic. The State Teachers College at Towson, Maryland offers two courses to students who are majoring in the teaching of mathematics. On the other hand, all undergraduates are required to study English, social studies, physical and biological sciences for 6, 12, or 18 semester hours. Hence prospective mathematics teachers have at least a background in the other subjects that the other students lack in mathematics. If the teachers of mathematics carry out a program of socialization with their own subject, they render a service not only to the furtherance of functional education but also to the enriched instruction of mathematical skills and meanings necessary for the preservation of our democratic society.

[2] National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education*. (Boston: Houghton Mifflin Co., 1923), p. 10.

One of the questions frequently asked concerning any core organization is whether or not mathematics is "in the core." That question was heard frequently during the Eight-Year Study[3] which provided inviting opportunities for curricular experimentation and it is heard today. The variety of answers given to this interesting question have been highly suggestive but satisfying to no one. Of course mathematics is "in the core" for if the core is regarded as "that aspect of the total curriculum which is basic for all students" it will, of necessity, include mathematics. Is there any school anywhere operating on the theory that the curricular experiences "basic for all students" should not include mathematics? Is anyone prepared to support the proposition that a man who is mathematically illiterate is appropriately educated for responsible citizenship in this atomic age? Any adequate study of "the major problems of living" calls for the use of mathematical concepts, principles and processes, and to discuss the question as to whether or not mathematics is "in the core" or "out of the core" is comparable to discussing the question as to whether or not language as a means of communication is "in the core" or "out of the core." Mathematics is not an inert body of knowledge to be moved "in" and "out" of a core at the whim of any given teacher or any group of teachers. It provides a language and a method which are essential to an intelligent study of any core problem.

There is no question as to whether or not mathematics is "in the core." The real question is the effectiveness with which mathematics can be learned through core procedures and that is a question which commands our careful consideration. One of the characteristics common to most all cores is a large block of time, usually from two to three hours, which provides flexibility in the planning of activities suited to the needs of the situation. It is within this block of time that learning activities "basic for all students . . . are organized without reference to conventional subject lines." Should mathematics be "in the core" in the sense that is to be taught within this block of time? Is mathematics best learned within such a framework? Fortunately, we do not have to depend on theory and unsupported opinion as we seek to answer these important questions for this problem has already been studied by interested people and their findings cannot be ignored.

Dr. Mumma in his survey of the core curriculum in the State of Maryland states:[4]

> It should be mentioned again that mathematics, science, music, and art are not usually included in the core but are most frequently taught as separate subjects.

Another study entitled "Opportunities for the Use of Arithmetic in an Activity Program" is reported by Paul Hanna.[5] Six teachers of grade 3 and six teachers of grade 6 were selected to assist in the study and in the seven schools represented by these twelve teachers "a strictly separate subject approach with a systematic course of study" was not followed. The children's experiences were not organized "with reference to conventional subject lines" but rather were organized and integrated around worthwhile interests.[6] "Each teacher was asked to record on prepared blanks every situation faced by individuals or by her entire class, in which there was a need for quantitative thinking and manipulation" and in this connection it should be carefully noted that the problems recorded were not to originate in a source external to the unit being studied but were to be "those prob-

[3] Wilford Aikin, *The Story of the Eight-Year-Study* (New York: Harper and Brothers, 1941.)

[4] Richard A. Mumma, "What Do Core Teachers Think of Core?" *The Maryland Teacher*. III (November, 1950), 26.

[5] Paul Hanna, "Opportunities for the Use of Arithmetic in an Activity Program," (*The Teaching of Arithmetic*, Tenth Year-book, National Council of Teachers of Mathematics [New York: Bureau of Publications, Teachers College,, Columbia Univ., 1935]), pp. 85–120.

[6] *Ibid.*, p. 90.

lems which arose in the pursuit of some child-selected activity."[7] These records were kept carefully over a period of four months and in the six third grades the total number of such quantitative problems which were essential "in the pursuit of some child-slected activity" was 234. Only 56% of these, however, called for actual computation which means that, on the average, only five computational problems per room per month were included. The "child-selected activity" in the six sixth grades yielded a total of 205 quantitative problems in 72% of which actual computation was needed. This means that if the arithmetic program of these children were limited to the quantitative work essential to the study of the core problem there would have been on the average approximately six computational problems per month for each of the sixth grade rooms. In view of these findings, it is not surprising that included in the report are the following statements.[8]

1. Functional experiences of childhood are alone not adequate to develop arithmetic skills.
2. The teacher should regard these meaningful arithmetic experiences as readiness and must provide sufficient periods of practice to assume mastery.
3. Some guide or criterion in addition to opportunities-found-in-activities is essential to the selection of arithmetic materials.

The report also emphasized the fact that "there is a large element of chance operating in the selection of units of work" and since the mathematics involved is determined by the type of unit selected it seems fair to say that any such educational procedure leaves to chance the mathematics studied.

Another interesting and significant study related to this problem was made by Catherine Williams.[9] While it is limited to one teacher and one grade in a single school the data were gathered over a nine year period and although "a single year was selected for the base year" the conclusions are derived from the evidence of nine successive years. The concept of the "experience curriculum" held by Miss Williams is also very significant. While it does not include "the cramping restrictions or expectations of predetemined organized subject matter requirements" neither is it to be "limited to a concept of curriculum based entirely on direct firsthand contacts with reality. It is not to be confused with a more or less casual opportunistic dependence on incidental learning" and the pupils "are guided into a realization of the ways in which meaningful learnings can be broadened, extended and applied."[10] Consistent with this philosophy Miss Williams skillfully uses the quantitative demands of a "core problem" to provide meaning and purpose for mathematics and the learnings thus acquired are "broadened, extended and applied." They are not limited to "those problems which arose in the pursuit of some child-selected activity" as was the case in the Hanna study and in this "extension" and "broadening" of learnings beyond the actual "opportunities-found-in-activities." It may be that Miss Williams is using the "guide or criterion," the finding of which was considered "essential" by the Hanna Committee when it emphasized the desirability of providing "sufficient periods of practice to assure mastery." This general principle seems to be supported by Miss Williams for in the weekly schedule of her students three 45-minute periods and two 30-minute periods are devoted to "mathematical experiences." Through such planned and creative guidance it is not surprising that the students participating in this problem consistently showed desirable growth in both reasoning and computational ability.

An interesting and significant study on the high school level is reported by Wal-

[7] *Ibid.*

[8] *Ibid.*, pp. 118–119.

[9] Catherine Williams, "The Contribution of an Experience Curriculum to Mathematical Learning in the Sixth Grade" (Unpublished doctoral dissertation, Ohio State University, 1947).

[10] *Ibid.*, p. 72.

lace P. Mannheimer.[11] He tells how mathematics entered the core curriculum at Forest Hills High School of New York in the spring of 1942. The three main subjects in this core were English 2, General Science 2, and Algebra 1. The first unit was "Health" and in the algebra classes certain very elementary statistical methods involving health data were used. Later, when the formula was studied applications were made to temperature conversion, multiplication of bacteria, and so on. Before the algebra classes completed their work on formulas, they began the next unit, "National Defense." Formulas were chosen, therefore, dealing with falling bodies, water and air pressure, velocities, forces on the airplane, and so on. The class then proceeded to a detailed study of the fundamental operations upon monomials and polynomials; and their connection with the core began to attenuate, except for occasional nods in its direction. Examinations were worded in distinctly core language, but the daily work of the classes had little to do with it. This estrangement persisted and at the end of the term Mr. Mannheimer felt that the "coring" of mathematics was fruitless. It was dropped from the core and still remains outside it.

Mr. Mannheimer felt that he had made sincere efforts to carry out the spirit of the core, and that he received the heartiest cooperation from his colleagues in the English and science departments. However, the difficulties were numerous. For example, in their statistical studies in the Health topic they found most of the data formidably complex. To discern trends in such data is a problem to tax an expert statistician, let along a class in Algebra 1. New concepts can best be learned in simple settings; but the core prevented their doing this. Eventually the instructor was obliged to throw the health figures overboard and substitute very simple data. When he was ready to return to the more complex picture, his colleagues were ready to leave it. In these topics the applications ranged far and wide; the core became a restraint rather than a stimulus to the motivation of algebra.

Some interesting observations as a result of his experience with the core were made by Mr. Mannheimer. He felt that every subject is, in its infancy, cored with others. Infantile learning, itself, is core learning, the core being the satisfaction of the infant's basic needs. However, when a subject is pursued further, it acquires an inner structure, and with this a most favorable order of topics and a definite tempo for teachability. Although no less useful in the world about us, it can best be learned by itself.

It appears that some subjects acquire this inner structure and resistance to coring earlier than others. For instance, Mr. Mannheimer suspects that spelling, almost from the start, can be learned most economically by concentration and drill. On the other hand, certain social sciences are successfully cored even in the high schools. The issue, he felt, reduces to this. Somewhere between infancy and college, mathematics takes the bit into its teeth and develops skittishness with respect to coring. He submits that this has already occurred before elementary algebra. If you try to core it with general science, its closest neighbor in the freshman year, a tug of war immediately arises; which order of topics shall dominate? Mathematics, he felt is almost inflexible; one cannot teach factoring before multiplication. Let us assume that we convince our science colleague of this and that he permits our order of topics to prevail in organizing the core. We cast about for a topic to suit formulas and equations. But science is, unfortunately, organized on different lines. The simple linear formula appears in diverse places, from a case of Ohm's law to the expression of Planck's hypothesis. Thorough coring with even the linear formula would require the most

[11] Wallace P. Mannheimer, "Mathematics in the Core Curriculum," *Highpoints*, XXVI (October 1944), 71–73.

acrobatic maneuvering through the halls of science on the part of our colleague. With other types of formulas the difficulties multiply. Let us consider equations. The linear equation is far too simple to portray certain elementary scientific data such as, say, the epidemic curve. On the other hand it is adequate in analysing, say, the Einstein photoelectric energy equation, a topic far beyond the scope of high school science. Mr. Mannheimer saw, then, that the two subjects, although indispensable to each other in the long run, had better not eat from the same plate. In fact, there is a real danger that attempts to "core" mathematics in this fashion may stress only its most superficial connections with the rest of the curriculum. "Does this mean that we must teach mathematics in an ivory tower?" Mr. Mannheimer asks. He answers:[12]

> Of course not! Observe the way we have enriched our students' understanding of the war effort through mathematics. But we have accepted these mathematical golden eggs without wreaking violence upon the goose. Let us, by all means, enrich our teaching of mathematics by relating it—far beyond the physical, biological, and social phenomena about us. Let us accept the obligation of the mathematics teacher to be a person of broad culture and quick perception, and not use any easy and artificial "core" as an escape from this responsibility.

These experimental situations serve to define a number of important problems concerning the effectiveness with which mathematics can be taught inside the core curriculum. Among these are the following:

1. How is the interest in the core problem affected if the students are required to learn with insight and understanding the mathematics essential to its study?
2. Can the core program provide opportunity for the development of mathematical understandings and skills commensurate with modern living?
3. If core activities are to be "organized without reference to conventional subject lines" what is the effect on mathematical learnings?

One of the attractive features claimed for the core curriculum is the opportunity which it provides for children to participate in the section of problems in which they have a real interest and about which they are genuinely concerned. The Hanna Committee, for example, refers to the "problems which arose in the pursuit of some child-selected activity" while Miss Williams reports that "active participation by the children in the selection of their unit of study was an integral part of the school's program of democratic living."[13] Interest is indeed a powerful factor in any educational program and every thoughtful teacher recognizes the value of plans and procedures which claim the interest of his students. This same important factor operates in the learning of mathematics and it might be well to consider what happens when the student's progress toward the solution of the core problem is deflected by the necessity for learning some mathematics along the way. Does his assumed interest in the core problem provide the needed motivation for a genuine understanding of the mathematics essential to its solution or is he tempted to look for answers regardless of how they are secured? When he is faced, for example, with the necessity of multiplying $1\frac{1}{4}$ and $3\frac{3}{8}$ or of finding the circumference of a circle with a given diameter, as was the case in the Hanna study, is he willing to submerge the larger drive stimulated by his interest in the core problem and make the kind of detour into the land of quantity and space which is essential if he is to have a real understanding of the processes by which these needed results are secured? Will he have the opportunity to use tangible materials and actually "see" with his hands and with his eyes what the product of two fractions really means before any effort is made to intellectualize the process? Will he be guided in an exploration of the relation between the circumference and diameter of a circle through which he finds an approximate value of pi? Will time be provided to acquaint him with the many significant episodes asso-

[12] *Ibid.*, p. 73.

[13] Williams, *op. cit.*, p. 29.

ciated with the evolution of this very important ratio or will his interest in the larger problem call so loudly for the needed answers that he will be encouraged to resort to meaningless manipulation to secure them? How can mathematical meanings be developed, important concepts clarified and general principles established in an atmosphere which places a heavy premium on the importance of answers?

Through such an emphasis the student will tend to believe that mathematics is nothing more than an incidental tool, irritating in its necessity and a heavy burden on the memory. In fact, there is more than a little truth in the statement of Mr. Mannheimer that in any such attempts to "core" mathematics we may be stressing only "its most superficial connections with the rest of the curriculum"[14] and to take the time needed for the building of real mathematical insight is seriously to interfere with the larger drive. A boy who really wants to go to the airport would be quite unhappy if he were first compelled to learn the operating principles of the automobile which took him there. If he does have a real interest in aviation, it is likely to be seriously dulled if, in pursuit of this interest, he is compelled to learn the operating principles of even the most elementary mathematics inherent in the development of this great industry.

The amount and kind of mathematics directly or even indirectly related to a core problem are variables with a wide range of values. The two most important factors which have direct influence on this situation are:

1. The nature of the problem selected for study.
2. The background of the teacher directing the study.

It is possible, for example, to develop an interesting and fruitful study of aviation without including any mathematics and that is precisely what was done by a teacher who himself had no mathematical background. His attitude toward this important field of knowledge does not differ greatly from that of many teachers who are providing very worthwhile experiences through the core program but who are quite unprepared to develop the meaning of even the most elementary mathematical concepts inherent in the study. On the other hand, the study by Miss Williams provides reliable evidence of what can be done by a teacher who recognizes the extent to which mathematical concepts permeate the fabric of modern culture. A less gifted teacher could have guided the students in a very satisfying study of "South America—Past and Present" with little, if any, emphasis on mathematics. Miss Williams, however, greatly enriched this unit through her own appreciation of the value of mathematical concepts and her skill in developing problem situations associated with these concepts. In the preparation of such materials, she used with considerable effectiveness ideas and topics related to South America but it should be emphasized that not all of the mathematics thus studied was essential to the unit and not all of it was "based entirely on direct firsthand contacts with reality." Her planning included definite provision for mathematical learnings and the usefulness of the five periods each week dedicated to "mathematical experiences" should not be overlooked. The activities of any core will inevitably reflect the interests and competencies of the teacher guiding the program, and there is no assurance that the core curriculum, as suggested by Professor Fehr,[15] will necessarily include "that mathematics which is needed by all members of a democracy for everyday intelligent evaluation of their lives." The core can be helpful in emphasizing the importance of mathematical concepts and principles and it is quite possible that core activities can provide a readiness for the study of these ideas. The daily schedule,

[14] Mannheimer, *op. cit.*, p. 73.

[15] Fehr, *op. cit.*, p. 7.

however, should include a special period dedicated to this purpose if meanings, insights, understandings and generalizations are to be made available for wide use. Mathematics teachers should of course, "contribute wisely and authoritatively to the proper execution of a core curriculum"[16] but there is likely to be a healthy difference of opinion as to just what is meant by its "proper execution." There are, in fact, large elements of chance operating in the selection and conduct of these core units and since an "intelligent evaluation" of responsible citizenship in our modern industrial democracy calls for mathematical literacy, will we be faithful to our obligations if we leave to chance the development of a mathematical program appropriate to general education?

The characteristic of the core which places even more serious limitations on its usefulness as an instrument for the effective teaching of mathematics is the demand that learning activities be organized "without reference to conventional subject lines." Mathematics is a system of ideas and to make it seem nothing more than a group of discrete manipulative skills, unrelated and unorganized, is to limit its usefulness and to distort its meaning. In a penetrating discussion of "Recent Trends in Learning Theory," Professor McConnell states:[17]

> In the case of arithmetic, attempts to psychologize the subject appear to have damaged it both logically and psychologically. By decomposing it into a multitude of relatively unrelated connections or facts, psychologists have mutilated it mathematically, and, at the same time by emphasizing or encouraging discreteness and specificity rather than relatedness and generalization, they have distorted it psychologically. They have obscured the systematic character of the subject, and have created a doubtful conception of how children learn it.

How can the potential learnings in a given situation ever become available for wider use unless they are organized around some unifying principle and generalizations established? Mr. McConnell further states that "as learning proceeds, one should construct more inclusive and systematic organizations of ideas and processes governed by the fundamental structure of the number system."[18] Surely no one would support the idea that unorganized experience is educative. What does one really learn from activity alone? The most active core classroom provides no greater guarantee of desirable learning than does any other kind of a classroom. Learning that is worth anything calls for organization of experience and if that experience is organized "with-out reference to conventional subject lines," it is very difficult to see how mathematical learnings can be expected. There are undoubtedly large values achieved through the organization of core activities "without reference to conventional subject lines" and the importance of these outcomes cannot be discounted. To assume, however that such procedures are effective in the learning of mathematics is to contradict the authority of experience. From the pen of John Dewey,[19] whose theories of education have exercised a large influence on the nature of the "core curriculum," comes the statement that:

> . . . it is as true of arithmetic as it is of poetry that in some place and at some time it ought to be a good to be appreciated on its own account—just as an enjoyable experience, in short. If it is not, then when the time and place comes for it to be used as a means or instrumentality, it will be in just that much handicapped. Never having been realized or appreciated for itself, one will miss something of its capacity as a resource for other ends . . . .

By what peculiar process of learning can mathematics ever come to be recognized as "a good to be appreciated on its own account" if it is decomposed into a "multitude of relatively unrelated connections

[16] *Ibid.*

[17] J. R. McConnell, "Recent Trends in Learning Theory," (*Arithmetic in General Education*, 16th Yearbook, National Council of Teachers of Mathematics, [New York: Bureau of Publications, Teachers College Columbia University, 1941]), p. 275.

[18] *Ibid.*, p. 273.

[19] John Dewey, *Democracy in Education* (New York: The Macmillan Company, 1930), p. 281.

or facts"? How can any student ever capitalize on its great "capacity as a resource for other ends" when he has been denied the generalization which make transfer possible? There is no assurance that mathematical skills used successfully in one situation will be available for use in another situation. Generalized understandings are needed and the development of mathematical understandings of this kind would be greatly impeded, if not altogether prevented when learnings are organized "without reference to conventional subject lines."

One of the great strengths of the core program is found in the classroom procedures associated with this curricular development. Pupil participation in the defining of problems, the gathering of relevant data, the organization and interpretation of these data as well as formulating and validating hypotheses are educational experiences in a very real sense. But these are procedures which may also be used with great effectiveness by teachers of mathematics and one way in which we can cooperate "in the proper execution of a core curriculum" is to provide for such activities in one's own classroom. Mathematics is in the "core" by virtue of the fact that it is "basic for all students." Let us teach it in such a manner that it is recognized by both students and faculty as "a good to be appreciated on its own account." Generalized understandings are essential if "its capacity as a resource for other ends" is ever to be appreciated and generalized understandings are the outgrowth of experience rather than the basis for it.

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## Change in Distribution of Yearbooks

**All the yearbooks** of the National Council of Teachers of Mathematics, which in the past have been sold by the Bureau of Publications, Columbia University, **may now be ordered directly from the Washington office** of the Council. Until January 1, 1953, yearbooks may be ordered from either the office of the Council or the Bureau of Publications. After January 1, the yearbooks will be available from the office of the National Council only.

# Use of Standardized Tests of Mathematics in New York State Secondary Schools

*By* Sherman N. Tinkelman *and* Minerva K. Chapman
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Probably the question asked most frequently of the New York State Education Department's test advisory service is: "What standardized tests are the other schools using and how well satisfied are they with their results?" To obtain a cross-sectional picture of actual practice in the state with respect to the use of standardized tests of mathematics, questionnaires were sent in the fall of 1951 to the approximately 1,300 schools of secondary grade (grades 7–12) in the state. The chairman of the mathematics department or the teacher in charge of mathematics in each school was asked to indicate the standardized tests of mathematics administered in the school during the 1950–1951 school year, the reason for using the tests, and his evaluation of the tests. Such information promises to be helpful to mathematics teachers in deriving suggestions for the evaluation and improvement of their own testing programs.

Considerable caution, of course, must be exercised in generalization from such a survey. The 508 schools responding to the questionnaire constitute only about 40 per cent of the 1,300 secondary schools in the state. The data may therefore reflect practice among schools interested in standardized tests more closely than they do practice among schools in general. As a group, too, these schools include a relatively larger proportion of upstate city and rural schools than of New York City schools, and relatively few non-public schools. Nevertheless, certain definite trends would seem to be indicated.

Even among schools answering the questionnaire, about 40 per cent made no use of standardized tests of mathematics during the 1950–1951 school year, although in most cases these schools were planning to administer such tests in the future. Failure to utilize standardized tests was otherwise attributable more to lack of any felt need than to actual dissatisfaction with or distrust of standardized tests.

Throughout the state in general the primary emphasis seemed to be upon tests of progress in arithmetic skills. Special prognosis tests in algebra and geometry were second in popularity. Achievement tests in higher mathematics were used much less frequently than the other two types, perhaps because most schools considered their needs in this area satisfied by the statewide Regents examination. There was a surprisingly wide variety of standardized tests used. The 299 schools administering tests used some 50 different tests during the school year. About 60 per cent of the schools, however, used no more than one test.

A striking feature of the questionnaire responses was the variability of practice among different types of schools. The New York City schools used relatively few tests and concentrated heavily upon their local arithmetic tests. Rural and semi-rural schools used a wide variety of tests, but concentrated heavily upon the test of arithmetic progress prepared by the State Education Department. Cities other than New York City showed wide range of tests and lack of concentration on any one instrument. The non-public schools were unique in their relatively greater use of achievement tests in higher mathematics, such as the Cooperative Test series.

For each test the schools were asked to indicate their evaluation on a three point scale—"recommend strongly," "recommend with reservations," or "do not recommend." Very few schools checked the

"do not recommend" column for any test. None of the ten most popular tests throughout the state was recommended strongly by more than two-thirds of its users and none failed to receive strong endorsement from at least one-third of its users, suggesting that the appropriateness of any specific test for an individual school may well be a function of the unique situation at that school. This point is best demonstrated by the fact that some schools reported excellent prediction in mathematics upon the basis of a standardized test, whereas other schools reported very poor prediction when using the very same test.

What were the reasons advanced by schools for withholding full endorsement of a test? Such reasons may indicate the nature of criteria applied by the schools in evaluating standardized tests for their purposes. About half of the schools criticizing tests were concerned with content. They felt that the tests did not correspond with their curriculum, were inappropriate in difficulty, failed to measure desirable objectives, or were insufficiently diagnostic. About one-fourth of the schools cited objections to mechanical features of the test, such as high cost, excessive time required for administration, difficulty of scoring, and lack of clarity of directions. The remaining one-fourth were dissatisfied with the results obtained, either because the scores failed to correlate well with other measures or because the norms were considered inappropriate.

Spring testing versus fall testing has been a matter of some controversy among the schools. Fall testing has been advocated in many quarters as a step designed to underscore the value of tests in instructional planning and to decrease the tendency to use test results for grading purposes or for evaluation of the teacher. The questionnnaire responses indicate that the schools seem fairly evenly divided on this question. Among the schools reporting the use of standardized tests, about one-third tested during September or October, about one-third during May or June, and the remaining one-third tested throughout the rest of the year. Despite the fact that one-third of the schools tested during May or June, only 16 per cent indicated that test results are used to determine pupil marks, in whole or in part. Among the 299 schools which administered standardized tests during the school year 1950–1951, the chief purpose, indicated by 65 per cent of the schools, was planning instruction on the basis of pupil needs.

To the school interested in initiating a program of standardized testing in mathematics, these data concerning actual practice may offer helpful suggestions. To the school already utilizing standardized tests the data offer an opportunity for comparison. In any case, however, the norm of standardized test usage throughout the state must not be interpreted as a standard for each school to achieve. The fact that a test is widely used does not necessarily imply that it is worth using. Indeed, so far as standardized testing is concerned, variability of practice and difference of opinion appear to be the outstanding feature among the secondary schools of New York State. What other schools are doing may therefore offer suggestions and ideas. What any one school should do will, in the last analysis, depend upon individual needs and results.

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In response to numerous requests, **reprints** have been prepared of the series of **twenty Signal Corps posters** which appeared in the May 1951 through April 1952 issues of THE MATHEMATICS TEACHER. The posters are printed on single sheets on one side only, ready for display on your classroom bulletin board. They show applications of mathematics in radio and communications. The entire set of 20 posters costs only 40¢. Send your order with remittance now to the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C.

# DEVICES FOR A MATHEMATICS LABORATORY

*Edited by* EMIL J. BERGER
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This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are requested to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning Devices for a Mathematics Laboratory to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

## A SLIDE RULE FOR QUADRATIC EQUATIONS

Klein in his book, *Elementary Mathematics from an Advanced Standpoint*, has a treatment of quadratic equations that suggested the laboratory device described in this article.

The solution of the quadratic equation $x^2+bx+c=0$ can be obtained graphically by finding the points of intersection of the parabola $y=x^2$ and the straight line $y=-bx-c$. The abscissas of these points are the desired roots of the given quadratic equation.

It is possible to find these roots mechanically by painting a permanent graph of $y=x^2$ on a piece of plywood and attaching a rotating and sliding bow to the back of the board in such a way that it will carry a string across the front of the graph (Fig. 1). By adjusting the bow the string can be made to represent any line with a slope of $-b$ and $y$-intercept of $-c$. In this way it is possible to represent any line of the form $y=-bx-c$.

Figure 2 is a top cross section view which illustrates how the bow and slider arrangement is constructed and attached to the back of the graph board. The segments $AC$, $BD$, and $CD$ represent the ends and back of the bow, and $AB$ is the string. Note that the back of the bow ($CD$) is free to rotate on the screw in the slider unless the wing nut is tightened, and also that the ends of the bow ($AC$ and $BD$) carry the string ($AB$) across the face of the graph board. The long stick which extends above and below the graph board in Figure 1 is the slider. Its long edges are bevelled to fit into a groove made of two strips bevelled in reverse fashion and nailed to the back of the graph

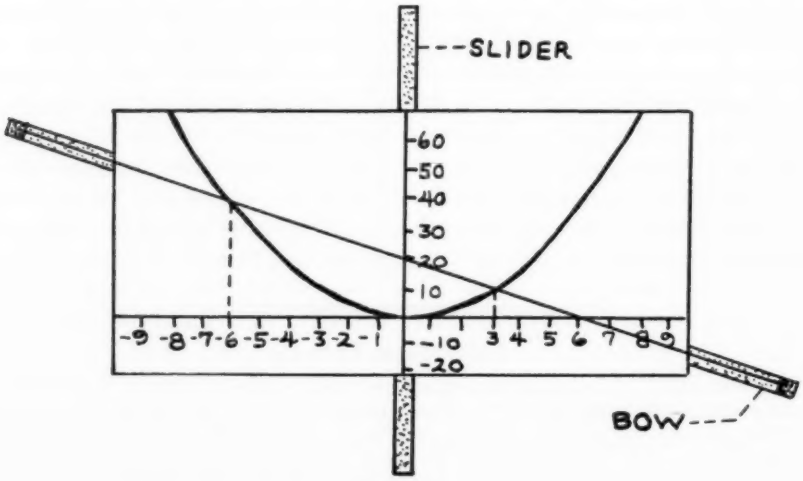


FIG. 1

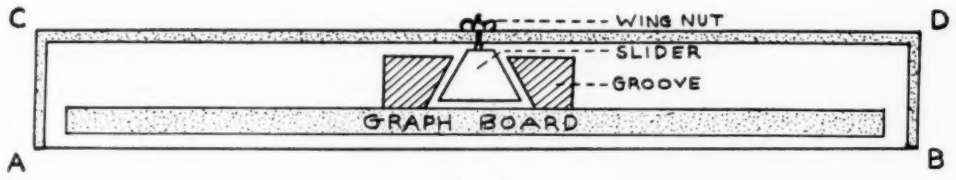


FIG. 2

board parallel to the $y$-axis.

The device illustrated in Figure 1 is set to solve the equation $x^2+3x-18=0$. The graph shows that the roots of this equation are $-6$ and $+3$. The following set of steps indicates a convenient way of obtaining the solution of any equation:

(1) Move the string line so that it passes through the origin.
(2) Rotate the bow until the line has a slope of $-b$.
(3) Clamp the bow to the slider by tightening the wing nut.
(4) Move the slider up or down to the intercept $-c$.
(5) Read the abscissas of the intersection points of the parabola and the straight line.

The reader may observe that the unit chosen for the $x$-axis is ten times as large as the unit on the $y$-axis. This makes it possible to read roots more accurately than would be possible otherwise. At the same time, however, setting the slope requires a little more care.

By using the device a teacher can assist students in developing their understanding of several interesting properties of quadratic equations. Consider, for example, the following ideas:

(1) The graph shows that when the intercept is positive (when $c$ is negative) there will always be two distinct roots.
   (a) How is this related to the discriminant test?
   (b) How is this related to the fact that $x_1 \cdot x_2 = c$?
(2) If the intercept is negative (when $c$ is positive) then there may be no real roots. If there are two real roots they will both be negative or both positive.
   (a) How is this related to the discriminant test?
   (b) How is this related to the fact that $x_1 \cdot x_2 = c$?

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## A DECIMAL PLACE VALUE DEVICE

Illustrated in this article is a device that can be used to assist pupils in visualizing the meaning of decimal place value. It was designed for use with junior high school pupils and can be used to explain problems from beginning work with decimal fractions to problems involving fractional parts of one per cent.

The device consists of a chart drawn on a sheet of plywood 15″×48″, ten rectangular wooden strips 1½″×15″, ten wooden squares 1½″×1½″, and ten small wooden rectangles 3/10″×3/4″. The following directions are offered to assist the teacher with the details of construction. Rule and paint the large piece of plywood as indicated in the diagram. (See Fig. 3.) To keep the ink from running with the grain of the wood spread a coat of shellac over the surface of the board before attempting the line work. Next outline the areas indicated as "red" and "yellow" with painters masking tape and paint these areas in one color at a time. When the yellow is dry paint in the black "decimal point." Finish the surface of the board with a coat of varnish and drive 3/4″ brads into the board at the points indicated in the diagram by small black dots.

Next cut ten plywood strips 1½″×15″, ten squares 1½″×1½″, and ten small strips 3/10″×3/4″. Paint all 30 pieces red and drill two 1/16″ holes in each at posi-

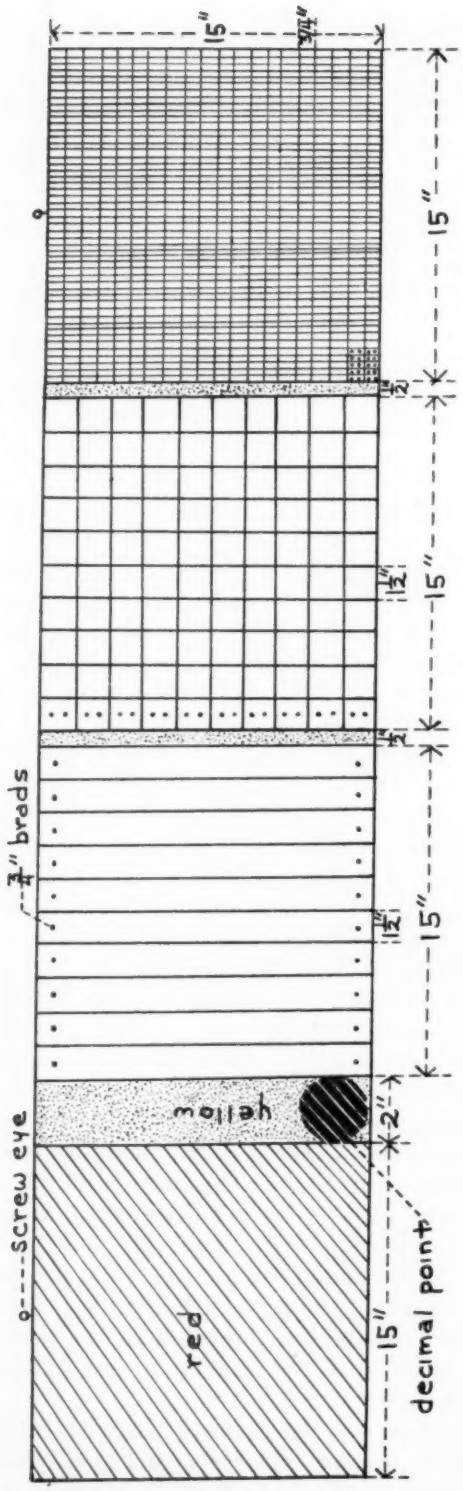


FIG. 3

tions calculated to correspond with those of the 3/4″ brads in the large board. (See Fig. 4.)

The board may be hung from two screw-eyes placed in its top edge. To keep the board from hanging perfectly plumb thus causing the small red pieces to fall off when placed on the brad hangers, a small

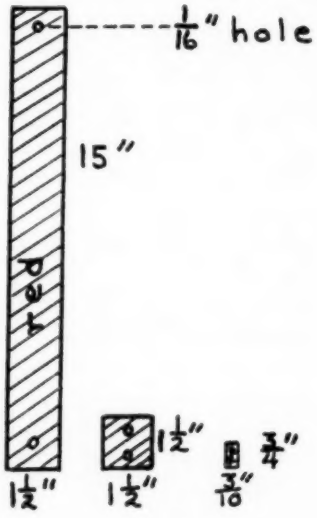


FIG. 4

block may be nailed to the back of the board along the bottom edge.

In using the chart the large red 15″ square at the extreme left of the board is assumed to represent unity. In accordance with this scheme each of the $1\frac{1}{2}''\times 15''$ strips then represents 1/10 of unity; each of the $1\frac{1}{2}''$ squares represents 1/100 of unity, and each of the small $3/10''\times 3/4''$ strips represents 1/1000 of unity. Thus the chart may be used to show that $1=10/10=100/100=1000/1000$, or $1=1.0=1.00=1.000$.

By placing one of each of the three different sized blocks in its appropriate place on the marked chart, the number 1.111 can be developed, as can any other three place decimal. In this way the pupil can be led to discover (or rediscover) the relation between the absolute value of a number and its position with respect to the decimal point. In the actual teaching situation it is also necessary of course to

develop the concept that this same relation exists no matter what quantity is taken as unity—e.g., one dollar, one square foot, etc.

The limitations of space make it impossible to describe in detail all of the many ways in which this board may be used to teach the meanings of decimals. One other seems worth mentioning, however. The problem of changing $\frac{1}{2}\%$ to a decimal ceases to be such a "problem" when this device is used. The pupil may readily be led to see that one-half of one of the pieces representing .01 is the same as five of the pieces representing .001, and that the answer must therefore be .005, rather than .5 or .05, or some other erroneous answer. Often merely pointing at the chart will provide the stimulus a pupil needs to recognize an error and correct his thinking.

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## Model of a Circular Cone With a Variable Axis

Probably one of the most useful models in the collection of student-made learning aids at Monroe High School in St. Paul is a model which helps clarify the definition of a circular cone. As usually defined a circular cone is a cone whose base is a circle. That circular cones may be either right or oblique is something which many students have difficulty in visualizing. Even after the definitions have been carefully illustrated with two dimensional drawings, invariably there are students who persist in believing that a cone can only be circular if a section formed by a plane perpendicular to the axis is a circle. That this is not necessarily true is a difficult matter to explain. The model described here was built especially to help students see that a section formed by a plane perpendicular to the axis of a circular cone may be an ellipse, and that every section of a circular cone formed by a plane parallel to the base is a circle.

The idea for this model came originally from a publication by Charles Elmer Rowe of the University of Texas. However the device described here is built somewhat differently and has some additional features [1]

The model consists of an axis rod mounted on a spindle, a base box with an open bottom, string elements strung from the top of the axis rod through a circle of holes in the top of the base box, and fish-line weights attached to the lower ends of the string elements.

In constructing the model it is probably best to build the base box first. Either $\frac{3}{8}''$ plywood or applebox lumber may be used for this purpose. The *inside* dimensions of this box are $6''\times6''\times5''$. If $\frac{3}{8}''$ plywood is used five pieces must be cut according to the following dimensions:

| | |
|---|---|
| 2 side pieces | $5\frac{3}{8}''\times6\frac{3}{4}''$ |
| 1 front piece | $5''\times6''$ |
| 1 back piece | $5''\times6''$ |
| 1 top piece | $6''\times6\frac{3}{4}''$ |

Drill a $\frac{3}{16}''$ hole in the top center of each side piece so that the centers of the holes are $\frac{7}{8}''$ from the top edges. (See Fig. 5.)

Next draw a circle with a diameter of $4\frac{3}{4}''$ in the center of the top piece. Space 16 holes each $\frac{3}{16}''$ in diameter around the circumference of this circle and cut a $1\frac{1}{4}''$ hole in its center. A nice job can be done on the large center hole by using a coping saw. The box may now be assembled, but do not fasten down the top until the spindle and axis rod have been assembled. Nails or screws will work equally well.

To build the spindle and axis rod assemblage illustrated in Figure 6 procure a $\frac{1}{4}''$ dowel 7" long and a $\frac{3}{4}''$ dowel 6" long. Drill four 5/16" holes side by side in the center of the $\frac{3}{4}''$ dowel and hollow out a slot as shown in the "Top View" in Figure 6. Fasten the $\frac{1}{4}''$ dowel in the spindle with

[1] Charles Elmer Rowe, *Basic Models for Engineering Drawing and Descriptive Geometry: The University of Texas Publication, No. 4343;* Austin, Texas: The University of Texas, November 15, 1943.

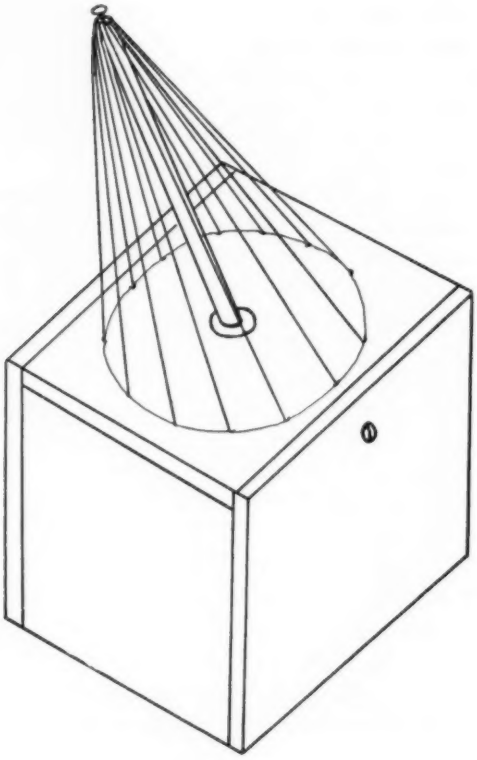

FIG. 5.

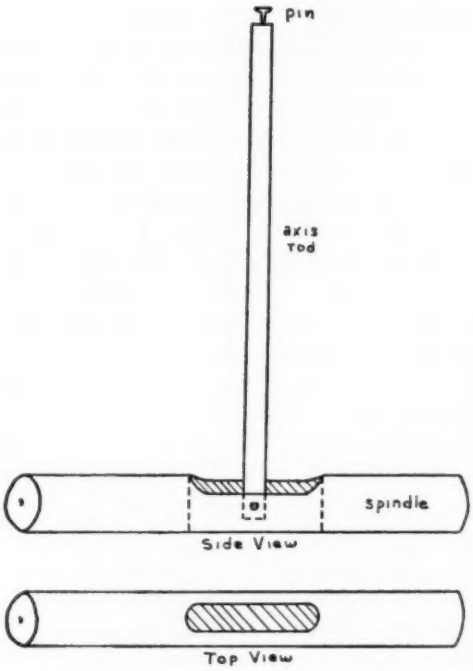


FIG. 6.

a *single* brad as shown in the "Side View." Drill $\frac{1}{8}''$ primer holes in both ends of the spindle, place it inside the open box and fasten it to the sides with screws which will turn easily in the 3/16″ holes previously drilled. Do not turn the screws down tight because the spindle must be free to turn inside the box.

Now fasten 16 strings to the top of the $\frac{1}{4}''$ dowel or "Axis Rod" with the aid of a short pin. Slip the large hole of the box top over the axis rod and pass the 16 strings through the $\frac{3}{8}''$ holes in the top. Cut the strings about 11″ long and fasten fishline weights at the lower ends so that the strings will hang taut. The box top can now be nailed down.

The nice thing about this model is that the axis may be inclined in any direction and rotated through 360° at any angle of obliquity. Students will enjoy this special feature of the model. Every section parallel to the top of the box (base of the cone) is a circle. It can easily be seen that when the axis is oblique to the base every section made by a plane perpendicular to the axis is an ellipse.

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# MATHEMATICAL RECREATIONS

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BEGINNING with this issue of THE MATHEMATICS TEACHER this department proposes to offer a series of problems and exercises which should lend to recreational activities in the classroom. Some of these problems may be found in books which are now out of print. Thus, their publication is, in the opinion of this department, justified. It is doubtful whether many of the classroom teachers have such sources handy. As far as possible, each issue will be devoted to a special topic. In this issue we will consider a series of problems which are associated with the number $\pi$.

1. The approximate numerical value of $\sqrt{2}$ is $1.41421\cdots$ and the approximate numerical value of $\sqrt{3}$ is $1.73205\cdots$. The following relation of the numerical values of $\sqrt{2}$ and $\sqrt{3}$ leads to some very interesting results.

$$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})=1.$$

But

$$\sqrt{3}+\sqrt{2}\approx 1.73205+1.41421\approx 3.14626.^{1}$$

Also

$$\sqrt{3}-\sqrt{2}\approx 1.73205-1.41421\approx 0.31384.$$

Thus $\sqrt{3}+\sqrt{2}$ gives the numerical value of $\pi$ correct to four significant digits, while $\sqrt{3}-\sqrt{2}$ gives the numerical value of $1/\pi$ correct to four significant digits also.

[1] The symbol "$\approx$" denotes approximate equality. The reader is counseled to be aware of the fact that the computations with the numerical values of irrationals and transcendentals (whether these computations are numerical or graphical) must conform with the rules of Approximate Computation. See: A. Bakst, *Approximate Computation.* (The Twelfth Yearbook of the National Council of Teachers of Mathematics [New York: Bureau of Publications, Teachers College, Columbia Univ., 1937.])

2. Are there any other pairs of square roots whose approximate numerical values will lead to the approximate numerical value of $\pi$? Here are some pairs of square roots whose differences give the numerical values of $\pi$ correct to three significant digits: $(\sqrt{15}-\sqrt{3})$, $(\sqrt{50}-\sqrt{14})$, $(\sqrt{79}-\sqrt{33})$, $(\sqrt{196}-\sqrt{118})$. No doubt, there are some other pairs. It would, probably, be an interesting activity, if some pupils were turned loose on a table of square roots and asked to discover such expressions. They might be asked to obtain the corresponding approximate numerical values of $1/\pi$ by adding the approximate numerical values of the square roots and by dividing the sums by the respective differences of the numbers under the radical signs.

In other words, such activities may be useful in connection with the studies of properties of $(a+b)(a-b)=a^2-b^2$.

3. The property of the approximation $\sqrt{3}+\sqrt{2}\approx\pi$ may be employed in connection with geometric constructions. We may obtain the approximate construction of the length of the circumference of a circle with a given radius as follows (see Fig. 1).

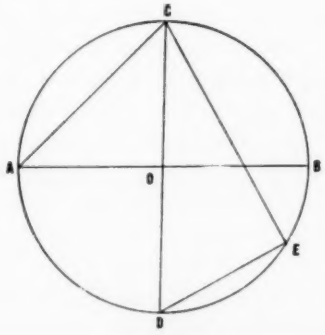


FIG. 1

Draw two perpendicular diameters $AB$ and $CD$ of the circle. From point $D$ mark off with the radius of the circle the point $E$. Join the points $A$ and $C$. Also join the points $C$ and $E$. Then $AC=\sqrt{2}R$ and $CE=\sqrt{3}R$. The sum of the straight line segments $2AC$ and $2CE$ will be approximately equal to the length of the circumference of the circle, correct to four significant digits.

4. If we assume that $(\sqrt{3}+\sqrt{2})$ and $(\sqrt{3}-\sqrt{2})$ are roots of a quadratic equation, we obtain this equation as

$$x^2-2\sqrt{3}x+1=0.$$

In other words, the roots of this equation are $\pi$ or $1/\pi$ correct to four significant digits. The equation $x^2+2\sqrt{3}x+1=0$ will have the roots $-\pi$ or $-1/\pi$ also correct to four significant digits.

Finally, the product of these two quadratic equations will be a biquadratic equation

$$x^4-10x+1=0$$

whose roots are: $\pi$, $-\pi$, $1/\pi$, or $-1/\pi$, all of them correct to four significant digits.

5. Another approximate construction of the length of the circumference of a circle may be obtained as follows (see Fig. 2).

Draw the diameter $AB$. At point $B$ draw the tangent $BC$ whose length is equal to $3R$ diminished by one third of the length of the side of an equilateral triangle inscribed in the circle. In other words,

$$BC=3R-\frac{R\sqrt{3}}{3}.$$

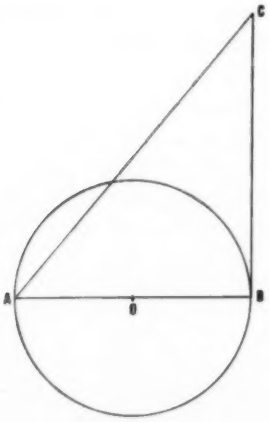


FIG. 2

Join $A$ and $C$. Then, from the right triangle $ABC$, we have that $AC^2=AB^2+BC^2$, or,

$$AC^2=4R^2+R^2\left(3-\frac{\sqrt{3}}{3}\right)^2$$

$$\approx R^2(13.333333-3.464102).$$

Finally, $AC\approx 3.14153R$, and the length of the circumference of the circle is approximately equal to $2AC$.

6. The numerical value of $\pi$ is often given in verse form, where the successive digits in its numerical value are represented by the number of letters in each word. The following are some of the verses.

A. Sir, I send a rhyme excelling
In sacred truth and rigid spelling;
Numerical sprites elucidate
For me the lexicon's dull weight
If Nature gain
Not you complain
Tho' Dr. Johnson fulminate.

B. How I wish I could recollect of circle round
The exact relation Archimedes unwound.

C. Yes, I have a number.

D. Now I know a spell unfailing
An artful charm, for tasks availing,
Intricate results entailing,°
Note in too exacting mood,
Poetry is pretty good,
Try the talisman. Let be
Adverse in gennity.

There are several similar German and French verses. The reader may find them in W. Lietzmann, *Lustiges und Merkwuerdiges von Zahlen und Formen* (Breslau: Ferdinand Hirt, 1929), pp. 19–20.

If the readers have any additional interesting problems with $\pi$, this department will welcome correspondence about them.

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## The Teaching of Trigonometry

FROM the earliest times until the beginning of the eighteenth century, trigonometry was regarded chiefly as the handmaiden to astronomy. Its development as a science may be said to have begun with Hipparchus about 150 B.C., but as a science it did not flourish with the Greeks. Not until some three centuries after Hipparchus did the Egyptian astronomer Ptolemy contribute to its advance by writing the famous *Almagest*. His influence upon trigonometry was destined to be felt for the next 1300 years, down to the time of Galileo. Throughout this entire period the solution of spherical triangles held the attention of mathematicians and astronomers. Much of the progress made from time to time was due chiefly to the Arabs and the Hindus.

The work of Peurbach, Rhaeticus and Regiomontanus, from about 1450 to 1550, marked the beginning of a new era in the development of trigonometry. The functions were now regarded as ratios rather than as line segments, and tables of values of the natural functions as we know them today made their first appearance. The invention of logarithms (about 1619) served both astronomy and trigonometry well. By this time, too, the surveyor's art had come to lean heavily upon trigonometry.

But the 17th century was to bring even more profound changes. The creation of analytic geometry and the calculus, together with algebra's coming of age, ushered in the era of modern mathematics. Trigonometry, too, became of age. Astronomy and surveying were relegated to subordinate roles, and with the work of de Moivre, Euler, the Bernoullis and their contemporaries, analytic trigonometry was born. It was not long thereafter that trigonometry found its rightful place in the realm of mathematical analysis.

If instruction in trigonometry is to be culturally stimulating and professionally useful, it must result in more than mere mechanical facility with trigonometric symbolism and manipulations. Really to understand trigonometry demands a certain degree of mathematical maturity. Superficially, it seems to be pretty simple stuff—a mixture of arithmetic, algebra and geometry. Actually, of course, the matter is far more subtle. The pupil must see old concepts through new eyes; he must come to regard angles, lines, ratios, variables and constants in unfamiliar and more general terms. Furthermore, he must become familiar with the notions of limit, directed lines, projections, and coordinates. The required synthesis is not altogether easy, either for pupil or teacher.

Whether trigonometry is presented as a separate course or interwoven with algebra and elementary analysis, the demands upon the teacher are not inconsiderable. He must teach trigonometry in the spirit of mathematical analysis. Much depends upon the organization of the material. In the past, computation, logarithms and the solution of triangles were perhaps overemphasized. It might be well to shift the emphasis and give more attention to coordinate systems, projections, transfor-

mations, vectors, the number system, radians and periodic functions. In short, we might teach trigonometry as a prelude to new horizons for the pupil in science, mechanics, electricity, optics, navigation and aviation, rather than simply as an handmaiden to surveying.


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# NOTES ON THE HISTORY OF MATHEMATICS

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## The Great Bet and When It Will Be Paid

The object of this article is to consider a quotation which, though not itself a problem, appears at the end of a set of miscellaneous examples in the Appendix to Roswell C. Smith's *Practical and Mental Arithmetic*.* The quotation concerns the presidential election of 1828.

As background to the quotation, one should consider that in the election of 1824, the candidates for the presidency represented the sections of the country rather than parties as such: John Quincy Adams of Massachusetts, William H. Crawford of Georgia, Andrew Jackson of Tennessee, and Henry Clay of Kentucky. No one of the candidates polled a majority of the electoral votes, Jackson's 99 exceeded the number of Adams who was next by fifteen votes, but 131 were required to elect. This threw the choice to the House of Representatives, and due in part to the fact that supporters of Clay and Crawford swung their influence to Adams, Adams was declared elected.

In 1828, the choice was between Adams and Jackson. The total number of electoral votes remained the same, 261. The campaign was a bitter one. The quotation reads:

> *The great bet, and when it will be paid.*—The public mind has been considerably amused, for a few days past, with a singular bet, said to have been made between a friend of Mr. Adams and a friend of Gen. Jackson, on the eastern shore of Maryland. The bet was, that the Jackson man should receive from the Adams man 1 cent for the first electoral vote that Jackson should receive over 130, 2 cents for the second, 4 for the third, and so on, doubling for every successive vote: and the Adams man was to have one hundred dollars if Jackson did not receive over 130 votes. According to the present appearances, Jackson will receive 173, 43 over 130, and the sum the Adams man will have to pay, in that event, will be $87960930222.07.
>
> But the joke does not appear to be all on the Jackson man's side. The money is to be counted, and it will take a pretty long lifetime of any common man to count out the 'shiners.' Let's see:—Allowing that a man can count sixty dollars a minute, and that he continues to count without ceasing, either to sleep, to take refreshment, or to keep the Sabbath, it will take him *twenty-seven hundred and eighty-nine years, nearly;* but allow him to work eight hours a day, and rest on the Sabbath, he will be occupied 9789+years; so that the Adams man, when he is called upon for the cash, may tell his Jackson friend, 'Sit down, sir; as soon as I can count the money, you shall have it; even the banks take time to count the money, you know."

The statement that "according to present appearances" the vote is 173 for Jackson, puts the date of the quotation between November 1828 and early February 1829. In 1828, there was no single election day for the whole country. Citizens of Pennsylvania and of Ohio cast their votes on October 31, elsewhere other dates were set the latest November 13 and 14 in Tennessee. In Delaware and South Carolina the old practice of having voting in the hands of the state legislatures continued. The votes of the electors were finally opened on February 11, 1829 and showed an electoral vote for Jackson of 178, not 173.

The allusion to the banks reminds one of the attack that Jackson was later to

*(Continued on page 454)*

* Roswell C. Smith, *Practical and Mental Arithmetic* (51st ed.; Hartford, 1842), p. 274. This text was first printed in 1826, with revisions and reprintings in 1827, 1829 and thereafter. Professor Karpinski in his *Bibliography* stated that the edition of 1834 was followed until 1845. It would be interesting to consult copies of 1829 when there were at least four editions to see whether this quotation appears in them.

# APPLICATIONS

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AN ARTICLE entitled "Concrete Interpretations of Directed Numbers" by Merton T. Goodrich in *School Science and Mathematics*, June, 1934, pp. 623–35, has been brought to our attention since our discussion of the methods of Mr. Schaaf (March and May, 1952) in the teaching of directed numbers. The article by Mr. Goodrich reflects a sensitivity to the heuristic, pupil-discovery style of teaching and in some ways is similar to some of the material previously presented in this department.

T.4 Gr. 10–12 *How Wet is the Earth?*

MR. KYLE SMITH of New York City submitted the following logarithmic computation of the number of gallons of water in the oceans. Without logarithms the computation might be of interest as low as the junior high.

First we have to compute the area of the earth's surface:

$$A = 4\pi r^2 = \pi d^2 = 3.14 \times d^2$$

where $A$ is the area in square feet, $r$ is the earth's radius in feet, and $d$ is the earth's diameter in feet

$$A = 3.14(7920 \times 5280)^2.$$

The Hydrographic Office has estimated that the average depth of the oceans is 2.36 miles or around 12,500 feet and that 70.8% of the earth's surface is covered by oceans. Assuming 7.48 gallons per cubic foot, the computation continues:

$Q$ (Gallons of sea water)
$= A \times 70.8/100 \times 12{,}500 \times 7.48.$

$\log Q = \log 3.14 + 2(\log 7920 + \log 5280) + \log 70.8 - \log 100 + \log 12{,}500 + \log 7.48.$

Using three-place accuracy throughout, $\log Q = 20.56049$.

Therefore, the number of gallons in Neptune's realm is 363 followed by 18 zeros. Why lose the opportunity here of using scientific notation? The answer may be written $363 \times 10^{18}$ gallons or, in the Clegg notation (February 1952), $3^{20}63$ gallons.

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T. 5 Gr. 10–12 *How Much Did Atlas Lift?*

Another problem submitted by Mr. Smith involving huge numbers concerns the computation of the weight of the earth in pounds. The controversial assumptions which he had to make before beginning his calculations were as follows.

The earth is a sphere having a diameter of 7920 miles. The mountains and oceans have been neglected as I have assumed that all the mountains have been sheared off at the earth's surface and spread over the oceans making the earth an approximate sphere. Furthermore some geologists maintain that the center of the earth consists of a gas under extremely high pressure. There is also the opinion that certain parts of the interior are in a molten state. Taking into account these varying conditions, textbooks on astronomy give the average density of the earth as 340 pounds per cubic foot, a figure which we shall use in the following computations.

$V$ (of sphere) $= 4/3\pi r^3$
$= 4/3 \times 1/8 \times 3.14 \times d^3.$
$V = .524 \times (7920 \times 5280)^3.$

$W$ (weight) $= .524 \times (7920 \times 5280)^3 \times 340.$

$\log W = \log .524 + 3(\log 7920 + \log 5280) + \log 340.$

$\log W = 25.11489.$

Therefore, the weight of the earth in pounds, assuming a hypothetical gravitational field of 32 feet per second per second, is 130 followed by 23 zeros, or $130 \times 10^{23}$ pounds or $1^{23}30$ gallons.

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Ar. 17 Gr. 9–12 *Per Cent of Water in Pigment*

Paint companies that use the flushing method of removing water from precipitated pigments need to know the per cent by weight of water remaining in the flushed product. This is done by dissolving 100 grams of pigment into a special flask containing toluene. By continuous boiling for several hours, the water is caught in a graduated tube, enabling one to read the cubic centimeters of water in the pigment. Let us assume the reading was 2.5 cc.

Now let one of your students compute the percentage from the above data. He will be surprised to learn that the percentage is also 2.5, because the cubic centimeters of water equals numerically the number of grams of water, and the grams of water per one hundred grams of pigment is equal to the desired per cent. This is true because the metric system is being used. There are many illustrations in laboratory work where a certain amount of an ingredient to be tested is taken in order that the final result can be used without further calculation.

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Ar. 18 Gr. 10–12 *Computing the Conversion Factor for Changing Specific Gravity to Pounds Per Gallon*

By definition: specific gravity (sp.g.) = grams per cubic centimeter.

Since 1 oz. = 28.35 grams,

$$\frac{\text{sp. g.}}{28.35} = \text{oz./cc.}$$

The reasoning behind this and subsequent steps is all-important in the development of skill in converting units. The reasoning of this step is somewhat as follows: since there are 28+ grams in every ounce, the number of 28's in the grams/cc is the number of ounces/cc. The student should be asked to reason through each step in this manner. Continuing, since 1 lb. = 16 oz.,

$$\frac{\text{sp. g.}}{28.35 \times 16} = \text{lbs./cc.}$$

Since 1 liter = 1000 cc.,

$$\frac{\text{sp. g.} \times 1000}{28.35 \times 16} = \text{lbs./liter.}$$

Since 1 gallon = 3.785 liters,

$$\frac{\text{sp. g.} \times 1000 \times 3.785}{28.35 \times 16} = \text{lbs./gal.}$$

The fraction can be computed to be 8.345, therefore:

$$\text{sp. g.} \times 8.345 = \text{lbs./gal.}$$

The above equation then becomes a method of converting rapidly from metric to English density units. This procedure is common in industries where the specific gravity of liquids is taken by laboratory technicians with hydrometers. The technician then uses this conversion factor to convert the metric readings to English units which are more meaningful to workmen in the factory. Whether one converts from grams directly to pounds or changes to ounces first is of far less importance than the technique of changing from one type of unit to another by appropriate conversion processes. The student should be provided only with the necessary data and the specific steps left up to him. You can find thousands of practice samples of conversion factors in any engineering handbook.

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Ar. 19 Gr. 7–9 *Some Amazing Speeds*

Here is a series of problems used by Dr. John Kinsella at the University School in 1943.

1. We think that 60 to 120 miles per hour in an automobile or even a souped-up hot

rod on a dirt track is fast and that 360 miles per hour or 6 miles per _____ _____ is only possible in a special type of car, yet think of these speeds:

2. The earth on a line running through Columbus, rotates on its axis 12.5 times as fast as a mile a minute or _____ miles per hour.
3. The earth whirls around the sun at the amazing rate of 20 miles per second or _____ miles per hour.
4. While the earth is on its journey, the moon is going around the earth at 40 miles per minute or _____ miles per hour. In a day the moon travels about _____ miles. Since the moon travels around the earth once in about 27 days, the length of its orbit or path is about _____ miles.
5. The moon is 239,000 miles away from the earth and the earth's diameter is 7920 miles. If you are real good in arithmetic, you will be able to compute the moon's orbit from this information and check it against your other answer above. Be sure to use the right radius for the moon's orbit.
6. The sun moves among the stars at a rate equal to about 65% of the speed with which the earth travels around the sun or _____ miles per hour.
7. The galaxy containing our sun with her nine planets is hurtling through the vast reaches of space at an estimated speed of 200 miles per second or _____ miles per day. This is about _____ times the speed of the earth (in number 4). You will agree this is going some, but where are we headed?

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### Professional Note

Following is a technique of mathematics pedagogy which has not been cited or validated to any great extent in the professional literature. It comes from Chapter 5 of *Gulliver's Travels, A Voyage to Laputa.* It appeared in "Mathematical Pie," October, 1951, a delightful little enrichment pamphlet published and distributed by the mathematics staff of Gateway School, Leicester, England.

> I was at the Mathematical School where the master taught his pupils after a method scarce imaginable to us in Europe.
>
> The propositions and demonstrations were fairly written on a thin wafer, with ink composed of a cephalic tincture. This, the student was to swallow upon a fasting stomach, and for three days following eat nothing but bread and water. As the wafer digested, the tincture mounted to his brain bearing the proposition along with it.
>
> But the success has not hitherto been answerable, partly by some error in the quantum or composition and partly by the perverseness of lads; to whom this bolus is so nauseous that they generally steal aside and discharge it upwards before it can operate; neither have they yet been persuaded to use as long an abstinence as the prescription requires.

Perhaps this is the origin of the educational conception that what a pupil really learns becomes a part of him.

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We would like to close this month's department with an appeal for information about a particular formula which we have heard is in use. This formula predicts the diminution of the area of healing wounds and was used in World War II in predicting early infections when wounds failed to heal as rapidly as the formula predicted. We would like to present this formula in this department with source references.

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## History of Mathematics

(*Continued from page 451*)

make on the Second United States Bank. The issue was doubtless in the mind of the friend of Mr. Adams.

The wager was made in Maryland, and its terms if analyzed show that had Jackson carried fewer than fourteen votes over 130, the payment would have been less than $100. In Maryland, the votes were more evenly balanced than in many of the other states. There were 25,527 for Adams and 24,565 for Jackson. If the bet was really made, it is certain that the Adams man had no expectation of a landslide for Jackson.

None of these considerations or data were included with the quotation; Roswell C. Smith makes no comment. He leaves the sum of the geometric progression and the attempt to comprehend the amount to speak for themselves.

# MATHEMATICS TESTS

*Edited by* JOHN H. HAYNES
*Acton High School, West Acton, Massachusetts*

## REVIEWS

**Lankton First Year Algebra Test.** Robert Lankton; Evaluation and Adjustment Series, Walter N. Durost, editor; World Book Co.; $2.15 per pack of 25; $.35 per copy; 1951; end of the year achievement; Forms AM and BM; 40 minutes working time; grade 9, first year algebra; "to measure the extent to which students have achieved the important objectives of a high school course in first year algebra."

*Description:* This test consists of 55 multiple-choice items which include such topics as vocabulary; manipulative skills as applied to equations, formulas, and other algebraic expressions; verbal problems of the type usually found in ninth grade algebra; and graphs.

*Validity:* Test items were built only after careful analysis of popular textbooks, courses of study, and reports of various groups on the objectives of first year algebra. Preliminary forms were given to about 2,000 students, and each item carefully analyzed. Item difficulty tables are included in the manual.

*Reliability:* Split-half reliability coefficients of .84 and .87 were obtained in two separate instances. An alternate form reliability of .81 was found for one group of students with less than one week between administrations. Further statistical work was done to show that in two out of three cases a student's "true" score does not differ from the test score by more than 4.8 point. Statistical work was also done to show that the two forms are almost exactly comparable in difficulty.

*Administration:* Specific directions to be read to the class are included in the manual. Working time is 40 minutes so that a 50 minute period should be ample. Separate answer sheets may be used. Scratch paper should be supplied.

*Scoring:* This test may be scored either by machine or by hand. A punched key is available for easy hand scoring if the printed answer sheets are used.

*Interpretation:* The raw scores obtained from the test are converted to standard scores which are connected with the Terman-McNemar IQ's of the standardization population. This also enables comparison of results with other tests of the series. Percentiles based on 3,183 students in 22 states are given for the standard scores. The test manual discusses how the results may be used for individual guidance, in evaluating individual achievement, and in evaluating group achievement.

*Conclusion:* This test seems to cover the range of material expected of a first-year algebra class quite well. One notes that in Form AM there is no problem dealing with the distance-rate-time situation which usually disturbs first-year algebra students. Plenty of thought questions of the other standard types are present, however. The test booklets are well printed and on the whole it appears to be an excellent test.

**Snader General Mathematics Test.** Daniel W. Snader; Evaluation and Adjustment Series, Walter N. Durost, editor; World Book Co.; $2.50 per pack of 25; $.35 per copy; 1951; end of the year achievement; Forms AM and BM; 40 minutes working time; grade 9; to measure achievement "of objectives of a course in general mathematics."

*Description:* This test in general mathematics consists of 65 multiple-choice items. The subject matter is made up of arithmetic (42%), informal geometry (23%), algebra (25%), graphs (8%), and numerical trigonometry (2%). Items test such topics as fractions, decimals, and per cent in arithmetic; formulas, graphs, and equations in algebra; area, volume, similar figures, and parallel lines in geometry; reading of graphs; and problem solving. The test was developed after examination of texts and courses of study used for general mathematics classes in junior high schools, and after consulting several statements of objectives for such general courses. The test booklets themselves are well printed in every way.

*Validity:* The test was administered in preliminary forms, each item carefully studied, and the final forms prepared. It is difficult in this type of test to say that it is valid for every general mathematics course since such courses differ very much. The test may be valid for the objectives set forth by the author, and for courses which have the same objectives and thus emphasize the topics covered by the test. However, any teacher of general mathematics should note the large amount of algebra and geometry which might make it a non-valid test for his particular course.

*Reliability:* Split-half coefficients of .80 and .84 were found for two cases. No alternate form reliability coefficient is given though the manual states that the forms are equivalent in that they treat in equal proportions the same topics and in that the item difficulty tables based on the preliminary forms show equal difficulty of the items.

*Administration:* As is the case of all tests of

this series, administration is very easy. Specific directions are given in the manual. Working time of 40 minutes means that a 50 minute period will give ample time. Scratch paper should be provided.

*Scoring:* This test may be scored either by machine or by hand. A punched key is provided for hand scoring.

*Interpretation:* As in other tests of this series, the raw scores are converted to standard scores which are related to the Terman-McNemar IQ's of the students on whom the test was standardized. Percentile norms which are based on 2,190 students in 22 states are given. The test may be used to evaluate individual achievement, to evaluate group achievement, or to base individual guidance. These uses are discussed by the manual.

*Conclusion:* Care must be used to make sure the class to be tested has had substantially the general mathematics course for which this test was constructed. This is not a test for a review arithmetic class, nor is it for groups of slow learners where emphasis is on the social aspects of mathematics. Likewise it was not constructed for the general mathematics course offered in the senior year by many schools. However, this test should fill a great need for schools which do offer this type of course in the curriculum of the ninth grade, for it is the only one of its type prepared for ninth grade general mathematics students.

**Shaycroft Plane Geometry Test.** Marion F. Shaycroft; Evaluation and Adjustment Series, Walter N. Durost, editor; World Book Co.; $2.50 per pack of 25; $.35 per copy; 1951; end of the year achievement; Forms AM and BM; 40 minutes working time; grades 10–11, plane geometry; "to measure the extent to which students have achieved the important objectives of a high school course in plane geometry."

*Description:* This achievement test is divided into two parts. Part A, which tests factual knowledge, consists of 10 true-false items, 20 matching type items in groups of 5, and 13 multiple-choice items. Part B, which attempts to measure the student's ability to use his geometric information, consists of one matching question with 5 items and 12 multiple choice items. According to the test manual, the test covers the following topics: fundamental concepts 7%, lines and rectilinear figures 47%, the circle 15%, proportions 22%, areas of polygons 5%, regular polygons 2%, and geometric reasoning 2%. This latter topic may seem to be lightly covered, but the author states it includes only analytic vs. synthetic proof, indirect proof, and converse of a theorem, rather than the entire use of geometric facts in problems and proofs. The test booklets are quite satisfactory, print being large and clear, and directions easy to follow.

*Validity:* Test items were developed only after exhaustive research of leading authorities in mathematics and in geometry in particular. The preliminary forms were given to about 2,200 students and the results carefully analyzed. Item difficulty tables are included in the manual. In fact much careful work seems to have been done to obtain a valid test for high school geometry.

*Reliability:* Both the split-half and alternate form methods gave a reliability coefficient of about .80. Further statistical work was done to show that there are two chances in three that a student's score on the test does not differ by more than 5 points from his hypothetical "true" score. The forms have been carefully tested to determine that they are almost exactly comparable even in terms of raw scores.

*Administration:* Specific directions to be read to the class are included in the manual. Working time is 40 minutes so that about a 50-minute period should be ample time to administer the test. Separate answer sheets may be used, and scratch paper should be supplied.

*Scoring:* The test may be scored either by machine or by hand. A punched key is supplied for easy hand scoring.

*Interpretation:* The raw scores of this test, as with others in this series, are converted to standard scores based on the median and standard deviation of the Terman-McNemar IQ Test for the group on whom the test was standardized. End of the year percentiles are based on the results from 2,914 students in 24 states. The test manual devotes considerable space to a discussion of how the test results can be used in evaluating individual achievement, in individual guidance, and in evaluating group achievement.

*Conclusion:* This test is worthy of serious consideration by anyone in need of an achievement test in plane geometry. The subject matter seems well covered, and a wealth of statistical work was done in developing the test. The items appear easier than on some of the older geometry tests. This is borne out by the fact that a raw score of 50% correct places an individual only in the 33rd percentile. Perhaps this is a good characteristic for the poorer student finds himself able to do enough not to become discouraged.

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**NEWS NOTE: The Mathematics Department of Kent State University** at Kent, Ohio, is holding its **Third Annual Fall Workshop** for high school teachers on Saturday, **October 18, 1952.** Two laboratory groups on construction of teaching devices will meet in the morning and seven groups will consider construction of tests at various grade levels during the afternoon period. Dr. Herschel Grime will speak at a luncheon on "Evaluation of Understanding in Mathematics." Copies of the program may be secured by writing to E. T. Stapleford, Workshop Chairman, Kent State University.

# WHAT IS GOING ON IN YOUR SCHOOL?

*Edited by*

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*and*

H. T. Karnes
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This Department will be continued under the co-editorship of H. T. Karnes, Department of Mathematics, Louisiana State University at Baton Rouge, and J. A. Brown, Wisconsin High School, Madison, Wisconsin. The plan for gathering materials will be similar to the procedure used in 1951–52. The co-editors will be happy to have *short* reports submitted by readers describing classroom procedures, data and related activities. Your suggestions for improvement of the Department will be welcome at all times.

## A High School Mathematics Club

Hardly had I gotten my second foot inside the school door before I was asked to help with the high school mathematics club. I was a young teacher eager to succeed in my new school position. If course I said I would "help." After assisting at one club meeting I was asked to sponsor the group myself.

It was fun but I was hardly prepared to accept the full responsibility. A nucleus of perhaps four or five students, left over from the previous term's club, was to form the core for my group. These students were definitely interested but all could not be counted on to attend every meeting. Except for these, very few pupils were eager to join. My experience in conducting a mathematics club was nil; material for use in club meetings was largely unknown to me. However, time brings success to one who couples imagination with experience. Today my club is a delight to me—perhaps the greatest satisfaction I have in my school hours. The club members are an unusually fine group of youngsters; their *esprit de corps* is a joy to witness. When, during a regular class period, one of them contributes information learned in club my chest expands at least two inches. However, I cannot claim I have done everything for the club members I should like to do. They lack the initiative along mathematical lines which I should love to see. Perhaps some day I shall have greater skill in developing that quality in them.

For the benefit of those who seek information concerning mathematics clubs I shall tell about mine. We meet after school on the first and third Fridays of each month. The club is headed by a president, vice president and secretary. When these officers are chosen at the beginning of the term the stipulation is made that the president and secretary be former club members. This requirement makes sure that at least two of the three officers shall know something of our usual procedures, thus promoting continuity from term to term.

As would be expected, the president conducts the club meetings and the secretary records the minutes. There is little other formality to our club periods since the roll is posted on the bulletin board near the door. Each pupil checks her name of the roll as she enters the room, thus saving time for more interesting activities. Each club program is planned about a week in advance by volunteers, the three officers and myself, as sponsor. This committee selects problems, puzzles and games which the pupils present.

Problems, as well as puzzles, are usually of a recreational nature. Here are a few examples:

1. Rover weighs 42 pounds plus a quarter of his weight. How many pounds does Rover weigh?

2. There are 8 green socks and 8 blue socks in a drawer. If you reach into the drawer in the dark what is the minimum number of socks you must take out before you are *sure* of having a pair that match?
3. Mrs. Powers is 24. She is twice as old as Patsy was when Mrs. Powers was as old as Patsy is now. How old is Patsy?
4. How can one put 10 lumps of sugar in three teacups so that each cup contains an odd number of lumps of sugar?
5. How can you take $\frac{1}{2}$ of 12 and get 7?

The problems we choose can usually be solved by algebra, arithmetic or simple logic. Occasionally someone presents one of a geometric nature. However, these must be simple since advanced freshmen may belong to our club. The club sponsor should have a variety of problems from which pupils may choose if the young people can't find any for themselves from outside sources.

At each meeting a "bonus problem" is presented. These problems must be done by algebra. No time limit is assigned to them—the pupil works on them in spare moments in the manner in which the radio Quiz Kids solve their extra problems. To get credit one must hand in a solution by the end of the club period. Credits of ten are given for each bonus problem correctly solved. At the end of the term a prize is given to the student with the most credits. Bonus problems at the beginning of the term are rather easy to solve. At the end of the term we select difficult ones so as to "break the ties." The club members take pride in being able to solve more bonus problems as they grow older and more experienced. Although most bonus problem prizes are won by seniors, occasionally a sophomore or junior will come out at the top.

Mathematical games can be in the form of relay races in which members are divided into teams. The teams have identical problems to solve and each member contributes some small part. These parts can be a series of fundamental operations performed in succession wherein each participant must use his predecessor's answer. For algebra relays signed numbers can be used or each person can be asked to solve an equation using for the constant term the answer given by the previous team member. In another game we have played, one pupil has a list of words which he reads, one at a time. The other members are required to think of a synonym for each work such that it will contain a number. The word "consumed," for example, is given. A synonym for it is "eaten." This word contains the number ten. "Solitary" has "*lone*" for a synonym. This word contains the number one.

Our mathematics club has produced short plays. One, consisting of scenes from "Alice in Wonderland" and "Through the Looking Glass" illustrating the use and abuse of logic, was given in assembly. Music to accompany the play was sung by a vocal group from the school.

At each club meeting a talk of about twenty minutes is given. Originally I prepared these but they were delivered by the young people. Experience showed such talks much less popular than those I presented myself due to the fact that the pupils lacked the power to forcefully express important points. However, frequently pupils help demonstrate parts of the talks I give. When a student delivers the whole talk it is on a subject in which she has done some research. Occasionally a graduate comes back to talk to the club members.

Since I have been giving so many of the talks and leading the ensuing discussions, I have a system to save time in preparations. I have made a list of more than sixty topics which I have found interesting to myself and to the club members. These topics I have listed in an index to a scrapbook. The scrapbook contains the notes and titles of references that I use. A large portfolio, similarly indexed, contains illustrative material suitable for the different topics. By having so many topics I need not repeat any within the three and a half year span which is the maximum time any pupil stays in the club. Dates

beside the topics tell when they were last used. Each time I give a talk I seek to bring the old outline up to date and to add new illustrative material.

Although historical topics are easy for a sponsor to prepare, I do not use too many of them. These are the titles to some of the topics on the history of mathematics which I discuss: "Numbers in Their Infancy," "Fun with Greek Mathematics" and "Archimedes' Experiment." Pupils seem to enjoy such discussions but only if they are a rarity and not the rule. Talks on mathematical symbols and devices such as Pascal's Triangle and Napier's Bones can be made fascinating.

One of the most popular topics I have found is "Mathematics from A to Z." In this discussion we go through the alphabetized topics from astronomy to zoology showing very briefly how mathematics is applied to these various subjects. Gifted pupils enjoy discussions on the connection between mathematics and other sciences and arts. Some of the topics I have discussed on such applications are "Mathematics in Architecture," "Mathematics in Music," "Astronomy and Mathematics" and "Physics and Mathematics." To supplement these discussions, trips by club members were made to the Franklin Museum and the Fels Planetarium.

Two topics that are favorites with club members are "When a Little Is a Lot" and its sequel "When a Lot Is A Little." The former is a discussion of the microscopic atom of which all things are composed while the latter is a discussion of the immense galaxies which make up the universe.

Equally fascinating to club members are talks on the various dimensions with special emphasis on the fourth dimension. The hypercube and the hypothetical feats that might be performed by a four-dimensional creature stir their imaginations.

Non-Euclidean geometry has a definite appeal to young people with a bent for mathematics. The theories of Riemann, Lobachevsky and Bolyai lead to some very interesting conclusions.

A general talk on logic with a presentation of the ludicrous conclusions into which false logic leads, amuses as well as informs. Pupils enjoy such examples as the proof that an elephant has seven tails.

No elephant has six tails.
One elephant has one more tail than no elephant.
Therefore, an elephant has seven tails.

"Great Oaks from Little Acorns Grow" introduces another subject which is sure to interest club members. This topic deals with the power of the exponent. The old story of the reward given to the inventor of the game of chess is new to many pupils. So also is the account of how a small sum of money, about eleven dollars, well invested at compound interest produced a fund sufficient to build thirteen bridges.

Other topics which have proved successful are "The Calendar is Dated," in which suggestions for improving the calendar are considered, "Numerology," "Prime Numbers," "Symmetry" and "Projections."

One club member, who had won a Westinghouse science award, told the other pupils about the machine she had constructed and her experiences on the Washington trip with the other award winners. Another gifted teenager, who had read, *Mathematical Snapshots* by Steinhaus, told about regular polyhedrons and explained how she had made several different ones that were collapsible. In a joint meeting with a mathematics club from another school several pupils explained a model to illustrate imaginary roots to an equation.[1]

Membership in our club is open to all pupils regardless of mathematical ability. However, we issue special invitations to

[1] Howard F. Fehr, "Graphical Representation of Complex Roots." (*Multi-sensory Aids in the Teaching of Mathematics*, 18th Yearbook, National Council of Teachers of Mathematics, [New York: Bureau of Publications, Teachers College, Columbia University, 1945]) pp. 133–34.

any students who have shown mathematical prowess in their regular courses.

Evelyn L. Owen
Girls High School
Philadelphia, Pennsylvania

### Industrial Survey

From April through May of this year, the Mathematics and Science clubs of Bedford High School conducted a survey of industries in and around Bedford. The purpose of this survey was primarily to get first-hand what industry expects of the high school graduate, especially in the fields of mathematics and science.

In the past five to ten years Bedford has expanded greatly from an industrial standpoint. This fact consequently places upon the schools of Bedford a responsibility. Industrial expansion requires educational expansion. Industries prefer to obtain from the local community the greater percentage of its personnel. Also a community should desire to keep at home a considerable number of its boys and girls after graduation from high school and college. This then requires an interest and understanding of what education can do for industry.

We feel that this survey was quite profitable in a number of ways. Here follows verbatim some of the statements made by key-men in the different industries.

Students should be urged to take all the mathematics available—algebra, geometry, and trigonometry. Applications of mathematics and shop mathematics should be studied thoroughly. Science, especially physics, both laboratory and lecture is important.

Algebra and geometry should be required of all students.

There should be three years of mathematics at least, more if possible.

All mathematics should be increased one year, especially geometry and trigonometry.

Our company has to reteach basic arithmetic such as decimals. Better form in problem solving should be stressed.

If I were planning a course of study, I would stress physics, chemistry, plane and solid geometry, four semesters of algebra and trigonometry.

The true importance of school is to teach and promote thought not knowledge: the kind of classes attended do not count as much as the amount of training the mind gets. Importance of education is found in learning to think and learning to live with oneself and others; as long as this is taken care of by high schools, colleges can build on it and produce the exact knowledge needed in following a career.

Mathematics develops a better reasoning ability and improves a person's power of concentration.

Our company expects a high school student to know how to use his head and be able to get along with others; also to take orders.

Greatest problem in all industry—How to think and use logic.

The survey added much to the interest and general knowledge of the individual students who took part. Twenty boys and girls participated in the study most of these visiting more than one industry. From one to two hours of time was spent in each establishment asking questions and touring the plants. The students had a few definite questions which they used in getting the information wanted, but in many cases the desired information was given without any questioning on the part of the students. Many comments were made on the willingness of those interviewed to answer questions and give information which might be of value and interest to the boys and girls. This comment made by one student was voiced by others—"It was a very interesting interview and I learned many things which I did not know before."

Reported from the *"Newsletter,"* Vol. III—No. 1 and 2, Sept.-Nov., 1951, published by the Indiana Council of Teachers of Mathematics. Editor, Olive Leskow, Gary, Indiana.

### A Project Giving Practice in Reading and in Writing Roman Numerals

Practice in reading and writing Roman numerals becomes a fascinating experience for junior high school pupils when the Roman numerals are used in original stories.

In this project each student writes a story using many different quantities

all expressed as Roman numerals. These stories are then typed, mimeographed, and made into story books.

The covers for these books are made by the pupils. Each child contributes a design in which Roman numerals have been used. One of these designs is then selected by the class and used for the cover.

Nowhere in the book is an Arabic numeral used. The table of contents, the page numbers, and the date of publication all are expressed as Roman numerals.

When the books are completed and each child receives a copy, the stories are used for oral reading. Needless to say, considerable interest is shown in this part of the project. The pupils carefully follow the oral reading as each member of the class takes his turn to select and read one of the stories. Although a pupil may make an error in reading a Roman numeral or in reading a word, no one is ever interrupted. Instead, all errors are discussed after the entire story has been read.

Typical of the stories written are the two which follow:

### The IV Deland Boys

*By* SHIRLEY MULLISON, Seventh Grade

The IV Deland boys were brothers. One was XV, one XIII, one X, and one IX. They lived at MDCL Beech Road in the year MCML. They lived in Utah on a farm with DLXXV acres. They had a pony which was IV years old, D chickens, and IV pigs. Every day they took a walk and always ate the right foods so they would be the healthy Deland boys.

### My Life

*By* DAVE BREIHOLZ, Seventh Grade

I was born in the Xth month and on the XXXth day. I have a dog named Spike. He is a collie and he is III years old, but in dog age he is about XXIV years old.

My dog was born in the XIth month and on the XIXth day. We have a little puppy now that is exactly LXXIX days old today.

I have lived in Ann Arbor for XII years. My oldest living relative is my great uncle. He is LXXXIV years old. He was born in MDCCCLXVII, just II years after the Civil War was over, which was in MDCCCLXV. My parents were married on August XXX and came to Ann Arbor in MCMXXXVII.

I was interested in scouting so I became a Cub Scout on September XXIV, MCMXLVII. (I was VIII years old.) I went through Cubs and then I became a Boy Scout in MCMLI.

ALICE M. HACH
Ann Arbor Junior High School
Ann Arbor, Michigan

### GEOMETRY FOOTBALL

It all came about when a co-worker expressed her sympathy for me because I taught mathematics, such a "dry subject and one which doesn't lend itself to creative planning." This was in the fall after I had attended the Mathematics Workshop at Louisiana State University so I was a little better prepared to meet her challenge than I might have been earlier. Over the weekend I thought up—in barest outline—this little game of football. Several football players were in my two classes which were finishing plane geometry. I presented the idea to them knowing full well that unless they "took to it" I should drop the whole thing. They liked it to the extent of asking me to meet a committee from the two classes the next morning at 7:15. From two such meetings the accompanying "Rules" were set up.

Teams in huddle to plan defense and offense. At stated time captains flip and teams line up. Ball is kicked. QB designates receiver. (Can't receive again for 3 problems.)

RECEIVER THEN—

1. Solves the problem—makes 15 yards and receives a "quickie"* (worth 1 point).

2. Fumbles—makes minor error. If seen by offense, costs 2 yards; if seen by defense, costs 4 yards. When referee alone sees it both teams are penalized. Only 1 fumble allowed.

3. Throws a pass—If offense can catch it, worth 10 yards. If not, defense must either catch it, making 8 yards or be penalized 15 yards for inability to solve own problem.

Delay of game..... 5 yards
Touch down.....32 yards

**When to Use:** Whenever the class reaches a set of more or less repetitious problems. We used it on Similar Triangles the first time.

**Why it Worked:** I was able to assign eight or even ten problems and receive smiles and enthusiasm whereas I often saw sober faces at a traditional two or three problem written assign-

* A "quickie" is a call for spelling, definition, postulate, or a review theorem. Any player who hasn't carried the ball may answer.

(*Continued on page 470*)

# AIDS TO TEACHING

*Edited by*

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*and*

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## BOOKLETS

*B. 104—Mathematical Needs of Prospective Students in the College of Engineering of the University of Illinois*

Office of Field Services, 309 Gregory Hall, University of Illinois, Urbana, Illinois.

Booklet; 6″×9″; 18 pages; 20 cents, each; 10 or more, 16 cents.

*Description:* This booklet reports a study conducted at the University of Illinois to determine the essential mathematical needs of students who expect to enter the College of Engineering of the University of Illinois. Since the criticism of the usefulness of mathematics was assumed to be a major one, data were sought concerning the utility of the concepts, principles and processes which were identified in a survey of the literature.

Data were obtained by interviews with engineering students and a questionnaire survey of instructors in the College of Engineering and the Department of Mathematics. High school mathematics teachers were also contacted to evaluate a list of topics. On the basis of the information obtained, the report lists the topics of secondary mathematics, an understanding of which is considered to be indispensable to the beginning engineering student if he is to begin his college mathematics with analytical geometry and finish the engineering curricula in four years. The bulletin suggests how teachers and advisors can use this list and how students can fulfill the admission requirements of the college. It gives brief but excellent suggestions for the improvement of the teaching of mathematics.

*Appraisal:* This check list will furnish the teacher with information for the guidance of students planning to enter engineering or related fields.

It emphasizes the need for better and more mathematics in the secondary school if we are to meet the needs of our technological society. However, it must be remembered that this list of topics is for a limited group and thus, does not mean that the high school mathematics curriculum should be built on these topics.

The careful reader will note that the report also places emphasis on broad objectives such as understandings, problem solving abilities, study habits, and arithmetic skills.

*B. 105—Without Pencil and Paper,* from *Study Arithmetics* (Books 3–6)

*B. 106—Without Pencil and Paper,* from *Mathematics and Life* (Books 1, 2 and 3)

Scott, Foresman and Company, 433 East Erie Street, Chicago 11, Illinois

Booklets; Each $5\frac{3}{4}$″×$8\frac{3}{4}$″; 14 pages; Free.

*Description of B. 105 and B. 106:* These two booklets contain material from the *Study Arithmetics* and the *Mathematics and Life* series of this publisher. They cover such topics as using your head, making change, rounding off numbers, comparing prices, and estimating distances. They are illustrated by cartoons. The first is aimed at intermediate grade teachers, the second at upper grade teachers.

*Appraisal of B. 105 and B. 106:* These helpful booklets are very attractive and are filled with valuable suggestions which should supplement and enrich the usual courses.

*B. 107—A Resource Unit for Teachers: Family Financial Security Education for Mathematics Students*

*B. 108—A List of Motion Pictures and Filmstrips on Financial Security*

*B. 109—Some Supplementary Teaching Aids on Financial Security*

The Committee on Family Financial Security Education, 488 Madison Avenue, New York 22, N. Y.

Mimeographed booklets; 8½″×11″; One side; Free.

*Description of B. 107:* (36 pages) There are statements of objectives; topics for the unit; a survey of the students' financial positions; 69 suggested activities for the students (with bibliographic references); teacher evaluation; tests on stocks, bonds and mortgages, and insurance; a bibliography of books, and of films and filmstrips; and a "pupil activity on banking."

*Appraisal of B.107:* In general this material is practical, unbiased, and logically organized. There are a few places where material seems to be poorly placed, but most teachers will welcome and be able to use this anywhere at the secondary school level of mathematics classes.

*Description of B. 108:* (15 pages) Here are materials classified under the topics of money management, banking, insurance, social security, investments, home ownership, and consumer economics.

*Description of B.109:* (16 pages) More materials, chiefly booklets, on the same topics as those covered in B. 108.

*Appraisal of B. 108 and B. 109:* These lists are excellent but should be obtained soon and used immediately, since such listings of free materials go out of date very quickly.

## CHARTS

*C. 37—Zippo Bar-Charts*

Zippo Bar-Charts, Jewell, Iowa.

Graph charts; Varied sizes from 25″×19″ to 8″×11½″; Black or red bars; $1.25 to $.40 depending on size.

*Description:* These charts enable one to make a bar graph quickly by tearing a perforated strip according to the length of bar desired. The charts are cross sectional graph sheets with space provided for titles and scales. The charts are supplied with or without scale numbers and in a variety of sizes and rulings. Bars may be shown horizontally or vertically, adjacent as a histogram or separated as a bar graph. The charts are made of two layers. Tearing off a strip of the top layer exposes a black or red layer which represents the bar of the graph.

*Appraisal:* These charts will furnish the teacher with an attractive, convenient method of making bar graphs. Large charts can be used to illustrate the construction and interpretation of bar graphs as well as furnishing a means of quickly recording and reporting the progress made by individual pupils for motivational purposes. The charts are too expensive for use by pupils.

*C. 38—Ten Years Growth in the United States*

Dun and Bradstreet, Inc., Advertising Department, 99 Church St., New York 8, N. Y.

Chart; 18″×28″; Color; 1950; Free.

*Description:* An outline map of the United States with state boundaries is surrounded by a bar graph. Red, green and black bars give the civil and business population and population density for each state. Data for 1940 are contrasted with 1950 figures. In each state pictures in color illustrate the principal products and industries of that state.

*Appraisal:* An attractive chart presenting important information.

*C. 39—Do You Know These Mathematical Words? How to Read Large Numbers*

Scott, Foresman and Company; 433 East Erie Street, Chicago 11, Illinois.

Chart; 18″×23″; 2 sides; Free.

*Description:* Twelve mathematics examples or expressions are shown and a vocabulary of important words is placed

beside the correct illustration. At the left is a glossary of the same words with formal definitions for reference. Such words as addend, decimal, fraction, divisor, numerator, remainder, and terms of a fraction are illustrated.

*Appraisal:* This is completely useful; nothing seems to be included which any teacher would care to omit. It is cold and formal in its appearance, so a few illustrations or decorations would help its attractiveness on a bulletin board, but there is no reason why other colors could not be used on the same bulletin board along with this. For teaching and reference purposes every mathematics room should have one of these up part of the time.

## EQUIPMENT

*E. 78—Addition Wheel*

*E. 79—Subtraction Wheel*

*E. 80— Multiplication Wheel*

*E. 81—Decimal Equivalent Wheel*

Air Sentry Co., 631 East Center St., Milwaukee, Wis.

$0.25 each, plus $0.10 postage on each order. Discounts on orders of 12 or more.

*Description of E. 78–E. 80:* Two cardboard circles mounted on an eight inch square cardboard can be used to find the answers to addition, subtraction, or multiplication problems for the combinations of all numbers from 1 to 10 inclusive.The circles are free to turn so that any combination of numbers can be made. When an example such as 3+4 is selected, the answer 7, is visible in a hole in the center circle. The wheels are made of durable, colorful cardboard.

*Description of E. 81:* A red cardboard circle $5\frac{3}{4}''$ in diameter is mounted on a $6\frac{1}{2}''$ green circle which is in turn mounted on a white square base. When the circles are turned so that fractions are formed on the circumference of the circles, the decimal equivalent of the fraction formed will appear in a hole in the other circle. The fractions which can be formed may have numerators from 1 through 10 and denominators 2, 3, 4, 5, 6, 8, 10, 12, 16, or 32. An appropriate game to build skill in converting fractions to decimals or vice-versa is described on the back of the number wheel.

*Appraisal of E. 78–E. 80:* These colorful devices will enable the user to quickly find the correct answer to number combinations without doing any thinking. Thus, they will be useful for pupils to use to check answers rather than to promote learning, retention, or the meaning of operations. The producer suggests that they be used to stimulate interest by using them as the base for games or contests.

*Appraisal of E. 81:* This number wheel will furnish the pupil an attractive gadget that will enable him to find the decimal equivalent of most common fractions. If it is used in reverse it will show the fraction equivalent to a decimal. The novelty of using these wheels should add interest to monotonous drill and encourage the checking of problems. The teacher will need to exercise care so that the method of conversion is not lost in the process of using the wheels.

*E. 82—Old Woman in the Shoe*

Ideal School Supply Company, 8322 Birkhoff Avenue, Chicago 20, Illinois.

Game; 9″×12″; $1.00. (Order Ideal School Supply Products from dealer nearest you.)

*Description:* This game consists of a picture of a family living in a shoe; there is a window in which the numbers 0 to 10 can be made to appear one at a time. Then there are 128 small cards ($1\frac{7}{8}''\times 2\frac{5}{8}''$); 64 contain addition facts without answers and pictures of children on the front, and the same fact with the answer on the back; there are 64 more small cards with subtraction facts. The number facts are used through the tens.

*Appraisal:* It is suggested that the answers to the cards be matched with the number appearing on the picture. This could be done individually or by teams. It is an interesting game to give variety to teaching, but is a little advanced for the first grade, for which it is suggested.

*E. 83—Numberland Special*

Ideal School Supply Company, 8322 Birkhoff Avenue, Chicago 20, Illinois.

Game; 9″×12″; $1.50. (See note on E. 82.)

*Description:* This game consists of a picture of a locomotive with a space in which the numbers 2 through 18 can be made to appear one at a time and 72 cards (3½″×4″) picturing railroad cars which show an addition or subtraction combination on each. The same problem appears on the back with the answer. The number facts are used through the eighteens.

*Appraisal:* An occasional use of this game should give variety to drill. The self-checking feature is useful. Three different ways to use the material are described on accompanying directions.

*E. 84—Parking Lot Game*

Ideal School Supply Company, 8322 Birkhoff Avenue, Chicago 4, Illinois.

Game; 12″×18″; $1.50. (See note on E. 82.)

*Description:* There are five "parking lots" which accommodate ten cars each. Each space in a lot is marked by a multiplication fact. Then there are 100 cards (1½″×2½″) with a picture of an automobile and a product on one side; the reverse side has a list of the combinations which give that product. Cars must be parked in the correct space in the parking lots. The five cards are graded in difficulty.

*Appraisal:* This game is suggested for the 4th, 5th and 6th grades and is probably quite suitable for them. There is just enough imagination in these games to make them appeal without being unduly complicated. There are enough good games for drill on the market now so that each school should be building an excellent collection.

*E. 85—Round-Up Game*

Ideal School Supply Company, 8322 Birkhoff Avenue, Chicago 20, Illinois.

Game; 12″×18″; $1.50. (See note on E. 82.)

*Description:* There are five "corrals" which accommodate 18 calves each. Each space in a corral is marked by a division fact. Then there are 90 cards (1½″×2½″) with a picture of a calf and a quotient on one side; the reverse side has a list of the combinations which give that quotient. Calves must be fitted into the correct space in the corral. The five cards are graded in difficulty.

*Appraisal:* Another good drill game. The only question is the amount of sophistication which many pupils bring with them to school now-a-days, and the doubt that games will be the correct approach to drill for all people. For some it will be fascinating, for others childish.

*E. 86—My Arithmetic Teacher*

The Arithmetic Clinic, 4502 Standford Street, Chevy Chase, Maryland.

Game; 6″×11½″; $1.15, 10% discount on a dozen.

*Description:* There is a cardboard base (5½″×9″) with two parts for illustrating addition and subtraction in one part and multiplication and division in the other. The numbers printed on it have holes associated with them and a collection of pegs (⅞″) can be placed in the holes. A four-page booklet (8½″×11″) gives detailed directions for using the device. Essentially this is just a way of arranging objects (pegs) systematically so that number facts can be learned.

*Appraisal:* The only advantages over the arrangements of sticks or circles on a desk top are the fact that this provides a systematic way of arranging them and also that the form of the printed number is immediately associated with the results of the arrangement. This latter is no small advantage but should certainly recommend the trial of the device. The directions are well written and well illustrated. This device is attractive and inexpensively priced.

*E. 87—Say It: Addition Game*

*E. 88—Say It: Subtraction Game*

*E. 89—Say It: Multiplication Game*

*E. 90—Say It: Division Game*

The Garrard Press, 119–123 West Park Avenue, Champaign, Illinois.

Games; $8\frac{1}{2}'' \times 10\frac{1}{2}''$; to schools \$1.25 each, \$4.50 per set of four.

*Description of E. 87 through E. 90:* Each game contains large cards, $8\frac{3}{8}'' \times 10\frac{3}{8}''$, (6 each in addition, subtraction, and multiplication and 5 in division) each containing 16 number facts. The cards are graded in difficulty. Then there are enough smaller cards, $2'' \times 2\frac{1}{4}''$, to correspond to the 96 or 80 number facts on the larger cards. The game is played like lotto with the smaller cards being claimed by stating the number fact and covering the one on the larger card. The first four in a line wins.

*Appraisal of E. 87 through E. 90:* This game is certainly interesting but less versatile than many drill games now being sold. It is greatly to be doubted how much will be learned from games where the answers are in sight. An exercise in reading "13 take away 7 is 6" will not do much toward strengthening that fact unless we consider such learning the building of automatic bonds to responses. Such a theory is dangerous. What assurance do these games give us that pupils will do any thinking about the answer at all?

## FILMS

*F. 71—Does It Matter What You Think*

British Information Service, 30 Rockefeller Plaza, New York 20, N. Y.

B&W (\$55 plus postage); 2 reels.

*Description:* The action in this film takes place at a weapons exhibit somewhere in England. Most of the persons that are discussed are the result of a conversation between several people who were attending the exhibit. The film shows the way in which opinions can be influenced and developed. It first discusses how the present creates a definite point of view. Books and magazines are used to illustrate the power of words in literature. Conversations in a factory illustrate how one person's opinion influences another. The film includes the part played by the latest movies, and political speeches in modeling attitudes. Its final thesis is to show how the organization of groups can add power to opinion in order to get done what they want done.

*Appraisal:* This is a war film produced in England for propaganda purposes. The photography is fair and the commentary definitely British. Although it is not mathematical in any way, it might be used to show the need for critical thinking in solving current problems rather than to accept biased statements of opinionated groups.

## FILMSTRIPS

*FS. 124–FS. 129—Adventures with Numbers*

*FS. 124—Making Change* (51 frames)

*FS. 125—Two-Figure Divisors* (50 frames)

*FS. 126—Zero in Multiplication* (43 frames)

*FS. 127—Meaning of Decimals* (44 frames)

*FS. 128—Dividing with Decimals* (50 frames)

*FS. 129—Dividing a Whole Number by a Fraction* (46 frames)

Popular Science Publishing Co., Audio-Visual Division, 353 Fourth Ave., New York 10, N. Y.

Webster Publishing Co., 1808 Washington Ave., St. Louis 3, Mo.

Color (\$6.00 each or \$31.50 per set of six strips).

*Description of FS. 124:* A trip to the store by Jack to buy groceries for his mother introduces the problem of making change for 88¢ out of \$1.00. The grocer

shows Jack how to make change by adding the cost of the items purchased, then adding coins until the sum is the amount given to the clerk. Selling food in the school store, selling packets of seeds in the community and illustrating purchases of candy, ice cream, a toy airplane, and a bat and ball furnish a variety of problems to practice this method of making change.

*Description of FS. 125:* This strip begins by quickly reviewing place value and division words. Clowns whose size represents 20 are used to illustrate division by 20. The number of clowns filling a gate whose sizes are 60, 67, and 83 show the quotients and remainders when 60, 67, and 83 are divided by 20. A clown of size 20 and his little brother of size 3 are used to represent the number 23. These are used to illustrate the meaning of quotients and remainders in the division of 69, 95 and 138 by 23. Several examples are given where the estimated quotient is too large. Oversize hats and dresses are used to emphasize the rule "if the quotient is too large, try the next smaller size." The strip summarizes by outlining the steps "divide", "multiply" and "compare" in every division example.

*Description of FS. 126:* This strip reviews the meaning of place value with a pocket chart. A variety of problems are illustrated to show zero as a place holder and as a multiplier. Short cuts in multiplying by even tens and in writing the partial products are discussed. The strip ends by summarizing and giving examples for the viewer to complete, some of which are complex enough to require paper and pencil computations.

*Description of FS. 127:* A cake, candy bar, and circle divided into 10 equal parts are used to show the meaning of tenths. Hundredths are illustrated by a pan of fudge and kitchen files, while thousandths are illustrated by two reams of paper. Cents, dimes, quarters and mills are used to relate decimals to money numbers. After diagramming the place value of whole numbers and decimals, practice is given in reading a variety of decimal numbers.

*Description of FS. 128:* Problems in buying 3 pairs of gloves for \$9.75, 8 skirts for \$7.60, and 4 candy bars for \$.28 and traveling 97.2 miles in three hours, introduces the method of dividing decimal numbers by whole numbers. After changing divisors and dividends by multiplying by 10, 100, or 1000 practice is given in the short cut method of multiplication. The problem of moving the decimal point in division is illustrated by representing the decimal point by Ike, Mike and Kit. These comic characters in different colors are then related to the carat commonly used by the pupil in placing the decimal point.

*Description of FS. 129:* Dividing a whole number by a fraction is illustrated by dividing a candy stick, two apples, two inches, four bricks of ice cream, two pies, two yards of ribbon, fudge, cake, seven yards of material, and seven pounds of cheese into fractional parts such as halves, thirds, fourths, $\frac{2}{3}$'s, $\frac{5}{6}$'s, $\frac{2}{5}$'s, and $\frac{3}{4}$'s. After showing that $2 \div \frac{1}{4}$ is solved by finding the number of fourths in one (4/1) and then multiplying by 2 to find the fourths in 2, the short cut of inverting the divisor and multiplying is illustrated. This inversion method follows the concrete illustration of the solution. The strip ends by giving several practice examples.

*Appraisal of FS. 124–FS. 129:* These filmstrips have excellent color and practical problems that should appeal to the young viewer. The illustrations of the meaning of division by fractions and two-figure divisors should clarify the processes and the answers obtained. A considerable proportion of the frames are given to the working of examples, but it is doubtful that subjects such as these can be presented without these abstract examples. At times unlikely problems are used, such as, finding the cost of one skirt if 8 cost \$7.60 or dividing 7 pounds of cheese into $\frac{3}{4}$ pound pieces. Some of the strips present a wide range of material, for example mills used to illustrate thousandths, or zero as a place holder and as a multiplier. This can be taken care of in projection by

showing only that part of a strip which is appropriate to the viewers. However, the value of a film or filmstrip in teaching an abstract process such as dividing by a decimal needs to be evaluated by research before it can be recommended as an effective aid to teaching.

*FS. 130—Taxes—Your City's Income*

Current Affairs Films, 18 East 41st Street, New York 17, N.Y.

B&W ($3.50); 33 frames.

*Description:* We are told that the city needs money for such services as police, schools, fire protection, street lights, health department, waste disposal, and parks and recreation. This expense is covered by a budget and raised through taxation.

*Appraisal:* The material is quite elementary but very logically presented. It should serve as excellent background for problems on taxation in consumer mathematics work.

*FS. 131—Pensions for All*

Current Affairs Films, 18 East 41st Street, New York 17, N. Y.

B&W ($3.50); 35 frames.

*Description:* The background of pension plans in the army and public service is given. A discussion of social security as a form of pension, and a discussion of industrial pensions are presented.

*Appraisal:* By extending our interests we can draw the whole curriculum into the mathematics department. It may well be that the subject of pensions, for many classes, will be so far from the path of the mathematics course that this excellent filmstrip cannot be used. It is up to each teacher to decide what particular applications will be fitted into the work. The presentation seems to be very well balanced and presents opposing viewpoints. It is already out-of-date with respect to prices.

*FS. 132—The Stock Exchange: Its Nature and Function*

*FS. 133—The Stock Exchange: How It Operates*

Current Affairs Films; 18 East 41st Street; New York 17, N. Y.

B&W ($3.50 each, both for $6.00).

*Description of FS. 132 and FS. 133:* Both of these filmstrips (using 32 and 33 frames respectively) show photographs to explain the stock market. In the first it is compared to a farmer's market where sellers and buyers can meet. Then reasons are given for having the stock market. In the second filmstrip it is shown how quotations are announced and how sales are made. The sale and purchase of stock for customers is followed through the exchange.

*Appraisal of FS. 132 and FS. 133:* As background for consumer problems these filmstrips are excellent although they contain little in the way of problem material. One criticism of basing problems in school on such situations is that pupils do not have the background to appreciate the problems—these filmstrips give that background. They both use a fairly mature vocabulary since they are intended to be used by adult groups as well as schools. The first justifies the stock exchange by giving reasons *for* it. Would not a true discussion filmstrip try to show the bad influences of the stock exchange as well? Parts of this filmstrip are very general and it is apt to express opinions without facts to back them up. The second filmstrip is quite detailed, but by being detailed is more convincing and more useful.

*FS. 134—Surveying: Measuring and Leveling* (#5–76)

*FS. 135—Surveying: Traversing* (#5–77)

*FS. 136—Surveying: Building and Utility Layout* (#5–78)

United World Films, Inc., 1445 Park Avenue, New York 29, N. Y.

B&W ($0.72 each, with 10% discount to educational institutions).

*Description of FS. 134:* In 58 frames

this filmstrip tells how to make horizontal measurements with the tape or chain, stressing the equipment used and the technique of handling it. It also tells how to make vertical measurements with the level, telling how to do differential leveling and profile leveling. It stresses the correct use of a notebook and ends with suggestions to avoid errors.

*Appraisal of FS. 134:* These filmstrips are some that were made for army training and should be ordered by the number following their titles above. The army approach to the problem stresses the "how" and minimizes the "why." If the "why" is supplied in school, the film will be an excellent aid. The appearance of soldiers and army installations may narrow the use for some; it may serve to point up the usefulness to others. The discussion of horizontal measurement is not too technical for high school use and is excellent in teaching that skill or for teaching appreciation of the types of applications made of geometry. The discussion of vertical measurement becomes a little too technical if appreciation alone is sought; however, all the details supplied need not be taught.

*Description of FS. 135:* In 71 frames this filmstrip tells how to use a transit, how to run a traverse by interior angles, how to prolong a straight line, how to run a traverse by deflection angles, and how to make a graphic adjustment of a traverse. It ends with suggestions for avoiding errors.

*Appraisal of FS. 135:* This filmstrip is too technical for the amount of skill usually developed in the use of the transit in high school mathematics courses, and it should not be used at all unless a transit of some sort is at hand to illustrate the ideas. Before discarding it for use, however, a teacher should consider carefully the tremendous amount of practical mathematics displayed. Must we always teach and analyze completely everything we use or can some be only illustration? The graphical adjustment of a survey is excellent to show how geometric constructions are used even if the class does not have the skill of making such adjustments.

*Description of FS. 136:* In 56 frames this filmstrip tells how to establish a base line, how to work from a base line, how to make maps, and how to lay out roads, utilities and buildings.

*Appraisal of FS. 136:* In general this filmstrip is too technical for mathematics classes for the amount of mathematics they would learn from it. The only usable feature is a number of illustrations of graphs.

*FS. 137—Proper Handling of Checks*

Young America Films, Inc., 18 East 41st Street, New York 17, N. Y.

Color ($5.50); 45 frames.

*Description:* In cartoon form this filmstrip shows how to make out checks correctly, how they should be checked by the person receiving them, the types of endorsements, how to make out deposit slips, how the bank collects the check, and how one should check his bank statements. This is one of a series of filmstrips on business education.

*Appraisal*: The presentation is clear and the material is all important in consumer education. It is interesting and not too technical for general mathematics classes, although the vocabulary is more suited to a high school than a junior high school level. The summary charts which show the path a check takes from people to banks and back again are excellent.

## INSTRUMENTS

*I. 36—Clock Face*

Ideal School Supply Company, 8322 Birkhoff Avenue, Chicago 20, Illinois.

Display instrument; $15\frac{1}{2}'' \times 15\frac{1}{2}''$; $2.00.

*Description:* A brightly colored clock face (11″ in diameter) is surrounded with illustrations from nursery rhymes, all on a sturdy composition board base. Two plastic hands (4″ and $5\frac{1}{2}''$) are mounted on a metal spindle.

*Appraisal:* Of course, clock faces for

teaching the telling of time can be made at home, but this device is so inexpensive, sturdy and attractive that it would certainly be preferred. There is every evidence of good workmanship and durability.

## MODELS

*M. 25—$H_2O$ Thermodynamic Surface* (#1710).

*M. 26—$CO_2$ Thermodynamic Surface* (#1710A).

W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago 10, Illinois.

Graphs; $24.00 each

*Description:* These plaster models are really three-dimensional graphs relating the variables pressure, volume and temperature for two gases. They are authentic, empirical relationships rather than the graphs of simple, mathematical equations. By keeping one of the three variables constant, families of curves connecting the other two variables can be drawn on the surfaces.

*Appraisal:* Three dimensional graphs need not be constructed on a secondary school level, but their meaning and reading can be explained. It would probably be better to start with cardboard or plaster models of simpler relationships between three variables, and then introduce the above models later. If the science department owns these, use them; if not, why not recommend their purchase.

*M. 27—Doodle Stix*

Dr. Isay Balinkin, University of Cincinnati, Cincinnati, Ohio.

Model-building materials; $1.00 per set, discount on lots of one dozen or more.

*Description:* The set seen by the reviewer consists of 12 sticks ($4\frac{1}{4}''$ long, $3/16''$ in diam.), 3 each of red, yellow, blue and green, and 8 rubber corner connectors which connect 3 sticks at right angles to each other. Since this connector is of rubber it can be distorted so that all three sticks can be in one plane and at a variety of angles to each other. Basically the parts are sufficient to build a cube, but they can easily be put in the form of a regular tetrahedron, square pyramid and other solids.

*Appraisal:* There are innumerable uses which mathematics can make of this material, especially if more than the basic set is available. Crystal forms can be built and changes in the volume of solid figures can be illustrated as the figures are distorted. These materials can be used also in discussing such questions as what solid figures are rigid, e.g., are all regular polyhedra, or are all those that have triangular faces rigid? The suggestion that letters and figures can be made of this material is of doubtful usefulness.

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## What Is Going On?

(*Continued from page 461*)

ment. It brought confidence to the weaker student. He was often able to receive a "pass" where he was unable to "see" at first. The "Quickie" was put in by the committee for these slower students. This carried no stigma because everyone had respect for our football player who came into the game just to kick the extra point.

**Additional Hints:** The huddle lasted about 10 min. Each captain had to be sure who could work which problems so as to avoid the penalty listed in the rules. The committee stressed limiting how often a student could handle the ball so as to keep it a class affair. The game points are optional. The ones here listed were those decided upon by the class.

**Other Uses:** I have found it to work on the stated problem assignments in algebra, too. With modifications, which most any class can provide, I'm sure other uses can be found. It wasn't used in place of traditional methods, but as a two-day assignment to add zest to necessarily repetitious work.

**Final Suggestion:** This, if used, should be presented in barest outline, allowing the students to build the game as they see fit. The ideas listed above to be used by the teacher only if she is asked for a point or two. My students did this; yours can do as well and better.

ODESSA DRAKE
Beaumont High School
Beaumont, Texas

# MATHEMATICAL MISCELLANEA

*Edited by* PHILLIP S. JONES
*University of Michigan, Ann Arbor, Michigan*

*56. Note on the SSS Case in Trigonometry*

WHILE BROWSING through the earliest French treatise on logarithms—*Traicté des logarithmes* (Paris, 1626) by Denis Henrion (c.1590–1640), Professor of Mathematics at Paris, my attention was caught by the method he adopted for the logarithmic solution of an oblique triangle when the three sides are given. The method is so ingenious that it must have been used many times before and since; yet today it seems to be overlooked entirely in favor of the effective but artificial method based upon formulas named for Henrion's contemporary at Oxford, Henry Briggs (1560/61–1630/31)—i.e., upon the tangent of half-angle formulas. Henrion's approach is as follows. Given the sides $a, b, c$ of a triangle with vertices $A, B, C$, drop an altitude $h$ from $C$ upon side $c$, and let $x$ and $y$ be the projections of $a$ and $b$ respectively upon side $c$ (extended if necessary). Applying the Pythagorean theorem to the two right triangles thus formed, one has $a^2=h^2+x^2$ and $b^2=h^2+y^2$. Subtracting the smaller from the larger, one obtains (assuming $a>b$) $a^2-b^2=x^2-y^2$ or $(a+b)(a-b)=(x+y)(x-y)$. Inasmuch as $x+y=c$, this can be written as $(a+b)(a-b)/c=x-y$, a form well adapted to the logarithmic calculation of $x-y$. With $x+y$ and $x-y$ both known, $x$ and $y$ are easily found by addition and subtraction. Then angles $A$ and $B$ are determined from $\cos A=y/b$ and $\cos B=x/a$. The method applies to all triangles, and angle $A$ is acute, right, or obtuse according as $y>0$ or $y=0$ or $y<0$.

The projection method of Henrion is less expeditious than the half-angle device of Briggs; seven logarithms and antilogarithms are needed to calculate an angle by the former method, as against five for the latter. The advantage of the latter is more pronounced when all of the angles are required. The chief merit of the projection device lies in the avoidance of artifice (although there is some slight advantage in the smaller number of intermediate operations required). Few students will long remember the formulas of Briggs, and fewer still will be able to reproduce the algebraic intricacies required for their derivation. Instructors who feel that formula memorization should be kept to a minimum, or that no formula should be used unless the student can supply its logical justification, may well find Henrion's method appropriate for their classes. Contemporary textbooks in trigonometry sometimes suggest a projection method as an alternative to the law of tangents in the SAS case; but an examination of well over a dozen recent textbooks has failed to uncover a similar alternative for the SSS case. Yet the method of projections is just as appropriate for the latter as for the former. In fact, the method has certain definite advantages where the given numerical values are too large for convenient application of the law of cosines, and yet too small for effective use of Briggs' labor-saving device. For example, given the triangle $a=45$, $b=23$, $c=34$, the angle $A$ (here deliberately chosen to illustrate the obtuse-angled case) is most easily found as follows. Let $x+y=34$. Then $x-y=(45+23)(45-23)/34=44$, whence $x=39$ and $y=-5$. Therefore $\cos A=-5/23$. Moreover, with no additional computation, one has also $\cos B=39/45=13/15$. That

is, one application of the projection method gives two angles, and hence all three are known—and this without the need to recall anything trigonometric beyond the definition of the cosine. Does not Henrion's procedure tempt one to try it in the classroom?

CARL B. BOYER
Brooklyn College
Brooklyn, New York

*57. Students Discover Mathematics*

The following three notes have for their common theme that students at all levels *can discover* mathematics for themselves, and that there is a real *thrill* for all involved when they do so.

We can not promise to print all notes of this type, but we would welcome other examples. We hope by printing a few examples that we may suggest some of the places and manners in which students may be led to make discoveries. Remember that, though the student makes the "discovery," his previous learning, his teacher, and the classroom atmosphere in which he lives are important factors in inducing in him the mental current which produces the discovery.

1919 Lake Street
EVANSTON ILLINOIS
May 21, 1952

DEAR MR. JONES:

Recently in my plane geometry class I put the enclosed Figure 1 on the board and asked the students to analyze it. My purpose was to have the students discover the length of the common external tangent. HAROLD N. WARD, a sophomore at Evanston Township High School, brought the enclosed answer to me the following day. This was a thrilling experience for me.

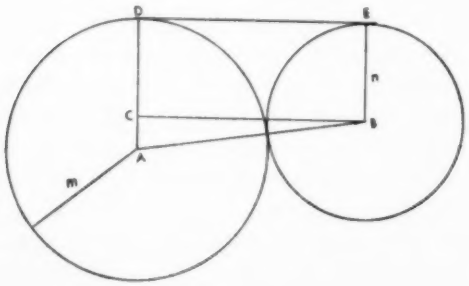


FIG. 1

Someone may have done this very thing before. But I am positive that for Harold it is original and exceedingly thrilling.

Very sincerely,
ROBERT HURST
Evanston Township High School

*A Geometrical Analogy to the Solution of the Diophantine Equation,* $a^2+b^2=c^2$.

Circles $A$ and $B$ are externally tangent, and $DE$ is a common external tangent. $BC$ is perpendicular to $AD$ from $B$. Since this is so, $AC=AD-BE$, or $AC=m-n$, obviously $AB=m+n$. Since $AB$ may be made any length, and likewise $AC$, then $ABC$ represents any right triangle. Now by Pythagoras' theorem $AB^2=AC^2+CB^2$, that is, $(m+n)^2=(m-n)^2+CB^2$. Upon expanding, $m^2+2mn+n^2=m^2-2mn+n^2+CB^2$, or $4mn=CB^2$. This, of course, results in $CB=2\sqrt{mn}$. Now, if we let $m=m_1^2$, and $n=n_1^2$, then $AB=m_1^2+n_1^2$, $AC=m_1^2-n_1^2$, and $CB=2mn$, which, of course, is the common formula and solution for the equation: $a^2+b^2=c^2$.

HAROLD N. WARD

*Editor's Question:* The perception of "analogies," or common elements, such as this between apparently diverse topics, is an essential element in teaching for *appreciation, meaning, insight,* and *understanding.* Can anyone see further reasons for what apparently, here, is an unmotivated parallelism in algebraic formulas? For further comments on Pythagorean numbers and a reference to their relation to a point lattice see THE MATHEMATICS TEACHER, Vol. XLV (April 1952), p. 269 ff.—P. S. J.

Stockton Street
HIGHTSTOWN, NEW JERSEY
March 28, 1952

DEAR EDITOR:

The eighth grade class of the Hightstown Higher Elementary section are studying "The Right Triangle." During our study I asked the group to learn the squares of all numbers from 1 through 20 and also the tens and fives from 20 to 100.

One of the boys, LYNN HUNT, age 13, returning to school the next morning said he had found an easy way to remember the numbers ending in 5. This is his method for numbers ending in 5:

a. I know the number will always have 25 on the end ($5\times5$ will always be 25—his reasoning).

b. I raise my ten number one figure higher.

c. Multiply the original number by the raised number and I have my answer.

Sample:

$25^2$; $3\times2$; 625

$75^2$; $7\times8$; 5625

("If I forget I don't have to multiply with pencil and paper.")

Later, he said, "I have a way to remember my teen numbers."

a. Multiply the figures in the ones column.

b. Add the number to be carried to tens place and also the number in ones column, as

$14^2$; $4\times4=$ 16
14
4
196 answer

$18^2$; $8\times8=$ 64
18
8
324 answer

Using the question method and knowing the family, I believe Lynn when he tells me he worked the methods out himself.

His first suggestion I have used with the group. They enjoyed working with Lynn's method. I can see no advantage in his second method because it taxes me (and also the group) to remember the process.

Yours very respectfully,
(Mrs.) Grace N. Rogers
Teacher of 7th and 8th Grade Mathematics
Hightstown, New Jersey

*Editor's Note:* For a further note on devices for squaring numbers ending in 5, see The Mathematics Teacher, Vol. XLIV (April 1951), p. 263 ff. Lynn Hunt's second rule may be generalized thus: *To multiply two numbers between 10 and 20, add the tens digit of one number to the other number, multiply this by 10 (i.e., add a zero to the product), add the product of the units digits.* In algebra this rule is: $(10+a)(10+b)=10(10+a+b)+ab$. It is, as Mrs. Rogers points out, an interesting teaching device for showing connections between arithmetic and algebra and thus "teaching for meaning," but *not* a fundamental part of that "functional competence" in mathematics which we hope to teach.—P. S. J.

Trinity High School
7574 West Division Street
River Forest, Illinois
May 12, 1952

Dear Dr. Jones:

Mention was made in a recent senior mathematics class that original proofs of the Pythagorean theorem have served as a pleasant hobby for a number of people. The next morning Mary Kay Egan handed in the accompanying proof. It was unsolicited and I feel sure it is "original" as far as she is concerned.

Truly yours,
Sister Mary

**Given:** Triangle $ABC$, with angle $C$ equal to 90 degrees. (Fig. 2)
**Prove:** $AC^2+BC^2=AB^2$.

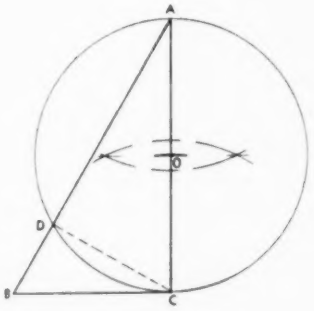


Fig. 2

**Proof**

| *Statements* | *Reasons* |
|---|---|
| 1. Construct circle $O$ tangent to $BC$ at $C$, with radius equal to $\frac{1}{2}AC$. | 1. At a point on a line, it is possible to construct a circle tangent to the line. |
| 2. Let $D$ be the point at which circle $O$ intersects $AB$; then $BC^2=AB(AB-AD)=AB^2-AB\cdot AD$. | 2. If a tangent and a secant are drawn to a circle from the same external point, the tangent is the mean proportional between the secant and its external segment. |
| 3. Draw $CD$. | 3. Two points determine a straight line. |
| 4. Triangle $BDC$ is similar to triangle $ABC$. | 4. If two triangles have two pairs of sides proportional, and the included angles equal, they are similar. |
| 5. Therefore $CD$ is perpendicular to $AB$. | 5. If two triangles are similar, corresponding angles are equal. |
| 6. Therefore triangle $ABC$ is similar to triangle $ADC$. | 6. If the altitude on the hypotenuse is drawn in a right triangle, the resulting triangles are similar to the original triangle. |
| 7. Therefore $AB/AC=AC/AD$, or $AB\cdot AD=AC^2$. | 7. Corresponding sides of similar triangles are proportional. |
| 8. Then $AB^2-AC^2=BC^2$. | 8. A quantity can be substituted for its equal. |

Mary Kay Egan

*58. New Slants on Old Problems*

I think the story is Stephen Leacock's. An inquiring stranger asks "What are you doing in algebra?" and the boy replies "I have finished clocks and am beginning work—'n-pipes." Some of the old problems dry up but do not blow away. There is a current belief[1] that we should teach them for what can be learned from them. Whether the teacher plans to make formal lessons of them or not, it is certain that he should study them for himself because they have a habit of bobbing up.

Take as a sample: Solve

$$x^2+y=7, \qquad y^2+x=11.$$

Some refugee from a geriatricist will innocently pass this to you remembering nothing about it except that it was hard when he could do it. He may give you the obvious set of solutions ($x=2$, $y=3$) to help you get started. There is no way of making the rest of the solution anything but tedious and that fact was made evident in journals[2] published about 50 years ago.

It is here suggested that you help your student by giving him apparently similar problems which he can solve readily; for instance:

$$x^2=y+2, \qquad x^2=2y+3, \qquad x^2=2y-1,$$
$$y^2=x+2. \qquad y^2=2x+3. \qquad y^2=2x-1.$$

He may be able to discover for himself why his proposed problem is perceptibly harder than these. If he likes to make graphs, the pictures will be found to support the algebra. If he is already studying analytic geometry he may be interested in trying to prove that the four common points of two parabolas whose axes are at right angles must always lie on a circle.

The problem of inscribing a rectangle of width 1 in a rectangle of dimensions 20 by 30, say, usually appears as a strip of carpet rolled out cornerwise on a ballroom floor. Some help is available[3] for setting up the equation (a quartic) but after that you are on your own. If, for example, one puts on a figure $x$ for the length sought and $A$ for one of the acute angles, these or similar equations come out at once:

$$\sin A+x\cos A=30,$$
$$\cos A+x\sin A=20.$$

Solve these for *sin A* and *cos A* and substitute in a well-known identity to get the algebraic equation which must be solved for $x$.

The problem of the ladders is a hardy perennial. Ladders of lengths 40 feet and 30 feet are placed across a street as shown in Figure 3. They are found to intersect

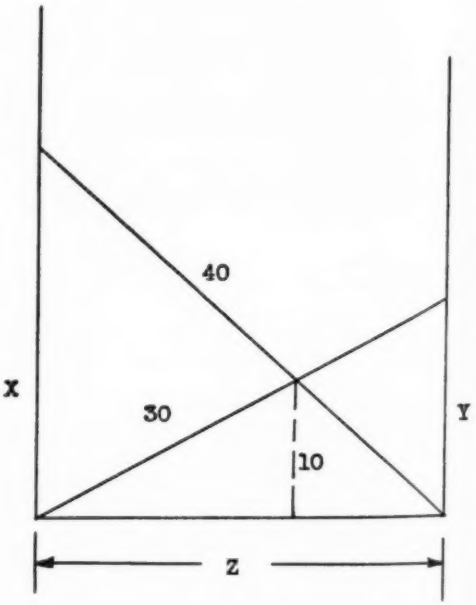


FIG. 3

[1] B. E. Meserve, "Let's Teach Angle Trisection," THE MATHEMATICS TEACHER, XLIV (Dec. 1951) pp. 547–550.

[2] Cajori on page 50 of his *Theory of Equations*, (1904), gives references to the *American Mathematical Monthly*, Vol. V, p. 291: vol. VI, p. 13; vol. VII, p. 169; vol. X, p. 192.

[3] A similar problem is solved by a method involving calculus on page 404 of Osgood's *Differential and Integral Calculus*, (1909).

The general problem of a rectangle inscribed in a rectangle is discussed in detail on pages 384–388 of vol. 1, part 2, of the Italian *Enciclopedia delle Matematiche Elementari*. (Milan: Ulrico Hoepli, reprinted in 1950).

at a point 10 feet above the ground. How wide is the street?

Suppose the longer ladder reaches the wall $x$ feet above the ground and the other $y$ feet above the ground. Let $z$ feet be the width of the street. Choose 10 feet as unit of length to simplify the computation.

Equating values of $z^2$, we have $x^2-y^2=16-9=7$. From similar triangles it is easy to find that $(1/x)+(1/y)=1$. From these we can eliminate one of the unknowns, by brute strength solve for the other finding the one positive root of a quartic equation, and go on to find $z$.

Let us instead observe that the two equations are equations of hyperbolas easily sketched and take a hint from that fact. Since we have

$$(x^2/7)-(y^2/7)=1,$$

let us introduce a new variable $\phi$ such that $x=\sqrt{7}\sec\phi$, and $y=\sqrt{7}\tan\phi$. The second equation will become

$$\cos\phi+\cot\phi=\sqrt{7}\approx 2.6457513.$$

It is easy by in-sinuation into tables of natural functions to find that $\phi$ is very close to 29.4022 degrees. From this by easy computations we find

$x\approx 3.03692$,

$z\approx 2.6033$,

$10z\approx 26.033$ feet. (*What we wanted.*)

Streets were narrower when this problem was young.

*Notes:* This method works with any possible numbers other than 40, 30, 10. If, for instance, we substitute 42, 33, 10, it works too well. The students should be grateful for an equation in $x$ which has a rational root, but he ought not to expect one. Making an equation which will have rational roots is an interesting pastime for the teacher and you will find some samples earlier in this paper, but the game too much resembles the kindergarten game: *Find the Peanut.* For kindergarteners it would be dirty mean not to hide at least one peanut, but makers of elementary textbooks should go to a good movie instead of piling up dozens of factorable forms and equations with rational roots and failing to mention that there are others.

NORMAN H. ANNING
University of Michigan
Ann Arbor, Michigan

*59. Pictures in Polars*

There are in the literature several examples of "dot" pictures to be traced by joining in sequence points whose coordinates have been given, and, on a

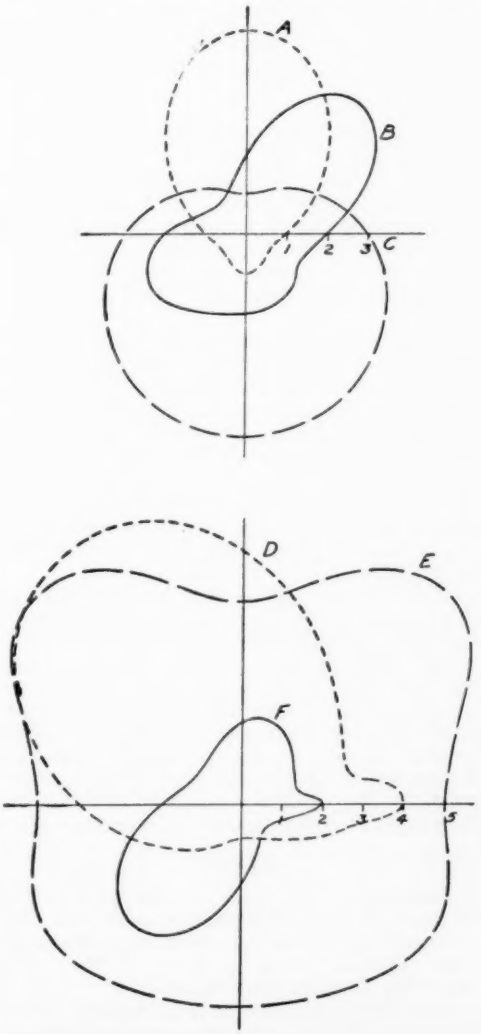


FIG. 4

(*Continued on page 491*)

# BOOK SECTION

*Edited by* JOSEPH STIPANOWICH

*Western Illinois State College, Macomb, Illinois*

## BOOKS RECEIVED

### High School

*Using Mathematics*, by David H. Patton, Superintendent of Schools, Syracuse, N. Y.; and William E. Young, State Education Department, Albany, N. Y. Cloth, xx+587 pages, 1952. Iroquois Publishing Co., Iroquois Bldg., Syracuse 2, N. Y. $2.52.

*Mathematics for Success*, by Mary A. Potter, Supervisor of Mathematics, Racine, Wis.; Anne L. Neitzel, Dorothy A. Root, and Frances C. Enright, all of the Washington Park High School, Racine, Wis. Cloth, viii+450 pages, 1952. Ginn and Co., Statler Bldg., Boston 17, Mass. $2.68.

*Everyday Mathematics*, by Harl R. Douglass, University of Colorado; Lucien B. Kinney, Stanford University; and Vincent Ruble, Jordan Junior High School, Palo Alto, Calif. Cloth, vii+504 pages, 1951. Henry Holt and Co., 257 Fourth Ave., New York 10, N. Y. $2.48.

*Making Algebra Plain*, by O. F. Revercomb. Paper, 256 pages, 1952. The McCormick-Mathers Publishing Co., Wichita, Kan. (Text-workbook, teachers manual, and tests). $1.80.

*Algebra, Its Big Ideas and Basic Skills*, Book II, by Daymond J. Aiken, Lockport Township High School, Lockport, Ill.; and Kenneth B. Henderson, University of Illinois. Cloth, xii+397 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $2.72.

*New Plane Geometry*, by A. M. Welchons and W. R. Krickenberger, The Arsenal Technical High School, Indianapolis, Ind. Cloth, vii+568 pages, 1952. Ginn and Co., Statler Bldg., Boston 17, Mass. $2.52.

*Dynamic Solid Geometry*, by David Skolnik, Central Commercial and Technical High School, Newark, N. J.; and Miles C. Hartley, University of Illinois. Cloth, viii+231 pages, 1952. D. Van Nostrand Co., 250 Fourth Ave., New York 3, N. Y. $2.48.

### College

*Algebra for Commerce and Liberal Arts*, by Alvin K. Bettinger and Wendell A. Dwyer both of Creighton University. Cloth, xi+225 pages, 1951. Pitman Publishing Corp., 2 West 45th St., New York 36, N. Y. $3.00.

*The Calculus, Arithmetic of the Age*, by G. P. Rawlings. Cloth, 84 pages, 1952. Percival Marshall and Co., 23 Great Queen St., London, W.C. 2, England. 10s.6d.

*Calculus*, by Atheron Hall Sprague, Amherst College. Cloth, xi+576 pages, 1952. Ronald Press Co., 15 E. 26th St., New York 10, N. Y. $6.50.

*Differential Equations* (Third ed.), by Max Morris and Orley E. Brown, both of Case Institute of Technology. Cloth, ix+341 pages, 1952. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y. $4.50.

*Differential Equations*, by Robert C. Yates, United States Military Academy. Cloth, vii+215 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $3.75.

*Elementary Differential Equations*, by Earl D. Rainville, University of Michigan. Cloth, xii+392 pages. 1952. The Macmillan Co., 60 Fifth Ave., New York 11, N. Y. $5.00.

*An Introduction to the Theory of Differential Equations*, by Walter Leighton, Washington University. Cloth, viii+174 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $3.50.

*Advanced Calculus*, by Wilfred Kaplan, University of Michigan. Cloth, xiii+679 pages, 1952. Addison-Wesley Press, Inc., Cambridge 42, Mass. $8.50.

*Elementary Analysis*, by Kenneth O. May, Carleton College. Cloth, xviii+635 pages, 1952. John Wiley and Sons, 440 Fourth Ave., New York 16, N. Y. $5.00.

*Elementary Coordinate Geometry*, by E. A. Maxwell, Queens' College, Cambridge. Cloth, 288 pages, 1952. Oxford University Press, 114 Fifth Ave., New York 11, N. Y., $3.50.

*Plane Trigonometry* (Third ed.), by Fred W. Sparks, Texas Technological College; and Paul K. Rees, Louisiana State University. Cloth, viii+199 pages, +tables, 1952. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y. $3.50.

*Plane Trigonometry*, by J. Topping, The Polytechnic, London. Cloth, viii+302 pages, 1952. Longmans, Green and Co., 55 Fifth Ave., New York, N. Y. $2.00.

*Mathematics for Finance and Accounting*, by J. B. Coleman, University of South Carolina; and William O. Rogers, Pennsylvania State College. Cloth, x+310 pages, 1949. Pitman Publishing Corp., 2 West 45th St., New York 36, N. Y. $4.00.

*The Theory of Matrices*, by Sam Perlis, Purdue University. Cloth, xiv+237 pages, 1952. Addison-Wesley Press, Inc., Cambridge 42, Mass. $5.50.

*Three Pearls of Number Theory*, by A. Y. Khinchin. Paper, 64 pages, 1952. Gravlock Press, P.O. Box 1223, Rochester 3, N. Y.

*Econometrics*, by Gerhard Tintner, Iowa State College of Agriculture and Mechanic Arts. Cloth, xiii+370 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $5.75.

*Advanced Statistical Methods in Biometric Research*, by C. Radhakrishna Rao, Indian Statistical Institute, Calcutta. Cloth, xvii+390 pages, 1952. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. $7.50.

*Calculus of Variations with Applications to Physics and Engineering*, by Robert Weinstock, Stanford University. Cloth, x+326 pages, 1952. McGraw-Hill Book Co., 330 West 42nd St., New York 36, N. Y. $6.50.

### Teaching of Mathematics

*Building Mathematical Concepts in the Elementary School*, by Peter L. Spencer, Claremont Graduate School, and Marguerite Brydegaard, San Diego State College. Cloth, ix+372 pages, 1952. Henry Holt and Co., 383 Madison Ave., New York 17, N. Y. $3.75.

*Instruments for the Enrichment of Secondary School Mathematics*, by Randolph Scott Gardner, New York State College for Teachers. Cloth, vi+98 pages, 1951. Edwards Brothers, Inc., Ann Arbor, Mich. Available from the Bookstore, New York State College, Albany, New York. $2.50.

*Illustrative Learning Experiences, University High School in Action*, by University High School Faculty, University of Minnesota, Emma M. Birkmaier, Ed. Paper, xiii+108 pages, 1952. The University of Minnesota Press, Minneapolis 14, Minn. $2.00.

*Guiding Children's Arithmetic Experiences*, by J. Allen Hickerson, New Haven State Teachers College. Cloth, xii+322 pages, 1952. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y. $5.00.

### Miscellaneous

*The Rational and the Superrational*, by Cassius J. Keyser, Columbia University. Cloth, viii+259 pages, 1952. Scripta Mathematica, Yeshiva University, New York, N. Y. $4.25.

*Work Measurement, New Principles and Procedures*, by Adam Abruzzi, Stevens Institute of Technology. Cloth, xviii+290 pages, 1952. Columbia University Press, 2960 Broadway, New York 27. N. Y. $6.00.

## REVIEWS

*Growth in Arithmetic Grade 3*, John R. Clark, Charlotte W. Junge, and Harold E. Moser. New York, World Book Company, 1952. v+314 pp., $2.12.

This is an attractive, carefully planned, well arranged text. Large print on good paper makes reading easy.

The Table of Contents is interestingly arranged in three columns, unit numbers, unit topics, and page numbers. Each unit requires only one line to name its topics, thus making it easy for the third grader to use.

First is a full page picture showing an interested class discussing "How We Use Arithmetic." The first Unit is "Telling Time and Using Coins." These are both interesting to the third grader and he can understand how arithmetic helps him tell the correct time and count his change. Each unit and subhead following is introduced very carefully from the children's fields of experience, and developed with the children in their activities and discussions.

There are just enough pages without pictures, designs, tables, illustrations, color spots, etc., to prevent monotony.

The entire book is developed with the idea of making arithmetic meaningful to the children and thus creating an interest in the subject and a desire to know more about it.—LORENA HOLDER, Dallas Public Schools, Dallas, Texas.

*General Mathematics for the Shop*, Gilbert D. Nelson, Frank C. Moore, Carl Hamburger, and Philip Becker (Editor). Boston, Houghton Mifflin Company, 1951. viii+440 pp., $2.64.

This book deals with applied mathematics for students in industrial arts. It presupposes a basic arithmetic knowledge.

The book is divided into three parts. Part 1 treats common fractions, decimal fractions and percentage. Part 2 contains applications of algebra and geometry including equation solving, metric geometry, constructions, and ratio and proportion. Part 3 deals with further applications of mathematics to cost of shop projects, power and transmission, screw threads, and tapers, and building construction.

Each new topic is introduced by directions readable by students, including pictorial and numerical examples. These are summarized in rules set off by heavy print. Immediately following is a group of problems called a "job" which starts with numerical problems and leads into applied problems.

At the end of each chapter is a summary, two review tests, and an arithmetic test which gives time and scoring scale for self testing.

Interspersed, where useful, throughout the book are tables including decimal equivalents, squares and square roots, sizes of screw threads, tangent ratios, ratios and amounts of ingredients for concrete, and tapers. In Appendix A are tables for drill and wire sizes.

In Appendix B are oral exercises designed to enable a student to improve his basic arithmetic. These involve addition and multiplication through ten, and reversals of the same sums and products in subtraction and division. No answers to problems are included in the book.—JOHN T. LADD, Highland Park High School, Highland Park, Michigan.

*Everyday Algebra, Intermediate Course*, William Betz and Alfred P. Windt. Boston, Ginn and Company, 1951. x+566 pp., $2.48.

This text for second year algebra, while unusually complete, is set up to meet the needs

of individual schools and students. For those students immediately continuing from first year algebra, Unit 1 and possibly Unit 2 may be omitted. For students who have had a lapse of time between elementary and intermediate algebra, the review of fundamentals is presented. Chapter tests may be used as inventory tests in deciding the needs of the class. The book presents a suggested schedule for time allotments in lessons for a two semester or one semester course. Exercises are well graded into "sets" making the text adaptable to average or superior students.

There are many excellent motivating devices. Between chapters appears a series of pictures and discussions presenting "The Role of Mathematics in the Modern World." Many chapters begin with interesting discussions of the material to be studied. Occasionally historical notes give background. Adequate use is made of the graph and the visual approach is applied and recommended where possible. Many problems relate to real-life situations. The techniques of problem solving and functional relationships receive much emphasis as does the systematic, gradual development of basic concepts, principles and skills.

Valuable to the student (and the teacher) should be the boxed summary at the end of each chapter listing what has been learned. Available also are chapter tests and occasional comprehensive review exercises. At the end of the text there is a general summary of aims of the course and a comprehensive mastery test.

It is the philosophy of the authors that "A second course in algebra is intended primarily for those students who intend to use it as a foundation for subsequent professional or vocational courses." With this type of student in mind, it appears that the authors have written a fine text for their purpose.—RUSSELL L. SCHNEIDER, Eastern High School, Lansing, Michigan.

*A Second Course in Algebra* (Second Ed., Enlarged), Walter W. Hart. Boston, D. C. Heath and Co., 1951. viii+488 pp., $2.40.

This textbook, as the title indicates, is an enlargment of Mr. Hart's *Second Course in Algebra*. Chapters 1 to 6 contain a review of the first course in algebra; chapters 7 to 13 contain the materials in a usual standard textbook in intermediate algebra. Chapters 14 to 17 and the Miscellaneous Topics contain material for a course in advanced or college algebra for secondary schools, teachers colleges or junior colleges. Some of these topics are arithmetic and geometric progressions, compound interest, synthetic division, solution of cubic and quartic equations, transformation of equations, permutations, combinations, determinants, an introduction to the calculus, etc.

Concerning the presentation of the material: (1) The instruction given is clear and full; (2) The vocabulary is simple and non-technical wherever possible; (3) There are many illustrative examples; (4) There are self-tests on each chapter placed at the back of the book; (5) Each new unit starts at the top of the page, usually a lefthand page, so that all instruction, illustrative examples, and practice appear on facing pages. This is particularly helpful to the teacher in giving individual instruction.

The book is well bound and the type-face used is pleasing to the eye. The illustrations are well chosen and skillfully placed.—MADELINE D. MESSNER, Abraham Clark High School, Roselle, New Jersey.

*Intermediate Algebra* (Second Ed.), Raymond W. Brink. New York, Appleton-Century-Crofts, Inc., 1951. xi+295 pp., $3.00.

This is a rather extensive text, easily adaptable to courses of varying lengths, for students with a background of one year of elementary algebra. Elementary topics are reviewed in a mature manner suitable for college students. The new beginning chapter on real numbers is well done for this level of instruction. Determinants are presented briefly in connection with the work on linear equations; determinants of the third order are expanded by the diagonal method only. The explanation of and exercises on principal roots and fractional exponents have been carefully written. Beyond the work on quadratics there are chapters on ratio, proportion and variation; progressions; logarithms; and the binomial theorem. The binomial theorem is stated for positive integral exponents; no proof is given of it. There is an abundance of good exercises, including adequate lists of applied problems, dispersed throughout the text. With the exception of a note in the late chapter on logarithms, there seems to be no attempt on the part of the author to develop a proper attitude toward and acceptable procedures for computation with approximate data. Rules applicable to the fundamental operations are given early in the text, e.g. the associative rule for addition. Throughout the text there appear many more rules for algebraic manipulations. For the mature student, the reviewer feels that these rules should be supplemented by remarks indicating their relation to the fundamental rules. For example, there is no mention of the rule that multiplication is distributive with respect to addition in the paragraphs on combining like terms, mutiplication of polynomials, and factoring.—LAWRENCE A. RINGENBERG, Eastern Illinois State College, Charleston, Illinois.

*Algebra by Visual Aids* (four vol. and answer book), G. Patrick Meredith; edited by Lancelot Hogben. London, George Allen and Unwin, Ltd., 1948. 550 pp.; 10 shillings; 8 shillings, 6 pence; 7 shillings, 6 pence; 9 shillings, 6 pence; and 6 shillings.

Here is an experiment in visual education through references to multi-colored charts (reminiscent of the triangle and solid numbers of Pythagoras) consisting of dots arranged in

patterns. These pleasing, colorful patterns are used to introduce from the start the excitement of discovery. This, having been encouraged leads to formal generalizations of rules. By bridging the gap between the particular and the general the visual aids help point out the time saving nature of the generalization and the place of the familiar and the particular as a check on the generalization.

Operating on the principle that it is generally easier to learn two skills successively than to acquire them simultaneously, they (the collaboration of the author and the editor has been so close that their respective contributions are not easy to detect) depart from the traditional order of presentation and concentrate first on the development of the new symbols and later upon operations with them.

The volumes are titled: (1) The Polynomials, (2) The Continuum, (3) The Laws of Calculation and, (4) Choice and Chance.

Book one deals with arithmetic progressions and related series. Book two reverses the natural order approaching the continuous function and graphical treatment of equations from the histogram of the arithmetic and harmonic series. The solutions of some equations are discussed. The theme of book three is geometric progressions. It ends with a chapter on logarithms. Book four treats of the binomial theorem as a basis of the algebraic theory of probability. Permutations and combinations are discussed.

To each of us interested in the teaching of mathematics, this radically different approach should have an appeal. By purchasing volume one, one can get an idea of the approach used in the separate volumes.—JOSEPH J. STIPANOWICH, Western Illinois State College, Macomb, Illinois.

*Teaching the Meaning of Arithmetic*, C. Newton Stokes. New York, Appleton-Century-Crofts, Inc., 1951. xi+531 pp., $4.50.

This is a timely book for teachers in service or in training on the theory and practice of teaching the meanings of arithmetic. Its principle aim is to give teachers a clear conception of what it means to do an effective piece of work in teaching number.

The question of meanings in the teaching of arithmetic has been given considerable attention in recent years. This book is devoted to the "Why," the "What," and the "How" in teaching the meanings of arithmetic. It contains a detailed discussion of the various steps in teaching the fundamental processes so as to secure better understanding and greater learning. Adding by endings and bridging are deferred until the child is ten years old or in terms of grades, the fifth grade. The treatment of division is especially commendable.

The author has brought together materials which deal with the psychological, the sociological, and pedagogical principles that underlie the effectiveness of a program of number study.—L. H. WHITCRAFT, Ball State Teachers College, Muncie, Indiana.

*Intermediate Algebra for Colleges*, J. B. Rosenbach and E. A. Whitman. Boston, Ginn & Company, 1951. x+219+answers, $3.00.

This text book is a typical brief traditional presentation of high school algebra, giving definitions, rules, principles, illustrative examples, and drill exercises. The fundamental aim of the presentation is to develop skill on the part of the student in manipulating algebraic expressions. The authors cite two purposes of the book: (1) as a terminal course; (2) as preparation for the study for college algebra. The contents include all those topics found in elementary and intermediate high school algebra textbooks.

The book is spotted with historical sketches that are pertinent to the material presented at these spots. A novel innovation is a set of WARNINGS inserted from time to time that tell the student to beware of false manipulations. Special attention, sadly missing in high school books, is given to the importance of equivalent equations and permissible operations on each member of an equation, as well as the nature of redudant and defective equations.

The lack of motivation and tie-up of algebra with concrete situations will not appeal to those interested in a meaningful approach to learning algebra. For those who desire only that their students be able to "do" their algebra the book will have great appeal. The distinction between a modern high school approach and this more traditional approach can be seen in the stress given here to transposition; rules of operation; cancellation; pure, affected, and complete quadratics; antecedent and consequent (instead of numerator and denominator); the use of D (for "varies as"); and graphing of functions by the location of points only.—HOWARD F. FEHR, Columbia University, New York.

*Algebra for Commerce and Liberal Arts*, Alvin K. Bettinger and Wendell A. Dwyer. New York, Pitman Publishing Corporation, 1951. xi+225 pp., $3.00.

More and more textbooks are being required for students who enter college without having studied intermediate algebra. In order to prepare students for a business-administration course or to provide algebraic background for trigonometry, this text has been devised. It bridges the gap between elementary algebra and mathematics of finance. A thorough review of elementary algebra is followed by chapters on exponents and radicals, including the binomial theorem, logarithms, the study of the quadratic equation, progressions, permutations, combinations and probability.

In general this book is designed for a course which covers the topics of intermediate algebra and then the topics from college algebra which have particular application to the mathematics of investment. Any student who has mastered the material in this small text should also be ready for more advanced courses in pure mathematics.—CAROLINE A. LESTER, New

York State College for Teachers, Albany, New York.

*Algebra: College Course* (Second Ed.), Raymond W. Brink. New York, Appleton-Century-Crofts, Inc., 1951. xiii+378 pp., $3.25.

This text supplies the material for a thorough and complete course in college algebra. Students who have had three semesters of high school algebra should be able to handle the content readily. The book is well organized and is adaptable to courses of different lengths and purposes.

An admirable feature of the book is the clarity of explanation of content. Examples to illustrate new content are used frequently and should prove to be very valuable to the learner. Plentiful and varied exercises follow the introduction of the new content and are graded in pairs with answers being given to the odd-numbered exercises. Optional material is offered and should be useful to the teacher in taking care of individual differences. The liberal number of applications that are thoughtfully distributed throughout the text add to its value. The problems offer a real challenge for the learner to apply his newly acquired knowledge.

Definite steps are taken in the text to help the learner organize his knowledge. This emphasis should result in a greater appreciation and understanding of logical thought by the student. The first chapter on numbers and operations will be referred to often by the student. The early introduction of the function concept and its use thereafter are other strong features of the text. Logarithms, inequalities, theory of equations, determinants, infinite series, and curve fitting are other topics that are well treated in the text.—WILLIAM L. CARTER.

*College Algebra*, Earle B. Miller and Robert M. Thrall. New York, The Ronald Press Co., 1950. xvii+493 pp., $3.75.

The authors intend this text to meet requirements of students who wish to make a life work of mathematics or some science which uses mathematics. To this end they have taken a middle course between a brief treatment of the various topics and one offering a proof for every statement.

Answers to the odd-numbered exercises are given; there are 63 pages of review topics; the text has a very good index. The treatment of matrices and determinants is one of the most complete this reviewer has seen in a beginning college algebra text. Since it covers so many topics, it is surprising to find no mention of ratio, proportion, and variation. The use of bold-faced type for important definitions, theorems, and formulas would have added greatly to the effectiveness of the book, because such a device helps the eye to find the formula desired and helps in applying the formula.

This text can be recommended for courses which are intended to give the students a good solid background for later work in mathematics.—REV. BONAVENTURE KNAEBEL, St. Meinrad Minor Seminary, St. Meinrad, Indiana.

*Life Insurance Mathematics*, Robert E. Larson and Erwin A. Gaumnitz. New York, John Wiley and Sons, Inc., 1951. vii+184 pp., $3.75.

Insurance is of such great importance to each one of us that I believe it should be taught to even a selected class of seniors in every high school in the land. This little book with all the recent major advances would be a happy choice for such a class and of course it would be an ideal first book for those college students with life insurance as their major field of study.

The book contains chapters on mortality tables, interest and annuities certain, life annuities, life insurance, net level reserves, advanced topics on fractional and increasing annuities and insurance, modified reserves, surrender values and gross premiums. There is also an appendix that shows an interesting relation between natural and net level premiums.

The advice to the reader, the solved examples which often give more than one solution, and the sets of problems all point to the conclusion that here is a book written by masters in the fields of both teaching and application.

Tables and answers to all of the problems are placed in the back of the book.—SAMUEL F. BIBB, Illinois Institute of Technology, Chicago, Illinois.

*Calculus* (Revised Ed.), Joseph Vance McKelvey. New York, Macmillan Company, 1951. vii+405 pp., $4.50.

In this revision, the text retains the same general design of the original edition. The changes include (a) The addition of an article on curvilinear motion and a chapter on the theory of limits; (b) general rewriting of the problem lists; and (c) a brief introduction to integration at the end of the chapter on differentials.

With the exception of the very brief introduction to integration in Chapter 6, the text rigorously separates the work in differentiation and integration. All topics usually found in a first year calculus text are included. The typography is good and the figures are well-drawn. The author's plan of illustrating new ideas first, before proceeding to formal definitions and theorems should aid the student in understanding new ideas.—CLEON C. RICHTMEYER, Central Michigan College of Education, Mt. Pleasant, Michigan.

*Essentials of College Algebra*, Joseph Rosenbach and Edwin Whitman. Boston, Ginn and Company, 1951. x+322 pp., $3.00.

*Essentials of College Algebra* has many of the excellent features of the three editions of College Algebra by the same authors. The explanations are very clear and concise, and the illustrative examples are adequate. Numerous written problems are included to test the stu-

dent's mastery of the principles studied. This is certainly a very desirable feature.

The lack of sufficient problems in factoring, preventing proper drill, and the omission of some types in the same subject is unfortunate. However, this need is partially remedied by the abundance of problems in fractions which require a knowledge of factoring.

With more emphasis on factoring, and an increase in the material on ratio and proportion, the binomial theorem, and theory of equations, this book might well be used in the traditional college algebra courses.—HERBERT L. LEE, University of Tennessee, Knoxville, Tennessee.

*Analytic Geometry* (Third ed.), W. A. Wilson and J. I. Tracey. Boston, D. C. Heath and Company, 1949. x+318 pp., $2.75.

Those mathematics teachers who have used the earlier editions of this excellent text will welcome this revision. Others will find that it strikes a happy medium between the complete geometries of the past and the modern summaries intended only as introductions to the calculus.

The Introduction contains a summary of algebra and trigonometry and the book closes with a complete list of formulas and equations developed in the text. Graphs of equations and equations of loci are treated in Chapter 2. General equations of conic sections are obtained by transformations of axes. Polar coordinates and tangents and normals are treated in separate chapters. There is a chapter on empirical equations and one on solid geometry.

The figures are well-chosen, complete and clear, and the format is most attractive. There is a wealth of problems; both the numerical drill type and those requiring geometrical proofs. The answers to the odd-numbered exercises are given and there is a good index.—FRANK C. GENTRY, University of New Mexico, Albuquerque, N.M.

*Analytic Geometry and Calculus*, Lyman M. Kells. New York, Prentice-Hall, Inc., 1950. viii+623 pp., $4.75.

The first two chapters (50 pages) of this well-written text are devoted entirely to the exposition of the fundamentals of analytic geometry. Thereafter, topics in analytic geometry are introduced wherever they are needed to provide the student with a background for the calculus theory. In the reviewer's opinion, this text may be used by those students who have already been exposed to a course in analytic geometry, as well as those students whose preparation consisted only of college algebra and trigonometry

Definitions of basic concepts such as limit, continuity, derivative, and the definite integral, are set down in italics and are preceded by well chosen and meaningful examples.

Integration is introduced on page 140, and the treatment of the definite integral is both lucid and mathematically sound.

Novel to this type of text is a chapter on vectors, which serves as a simple introduction to vector analysis.

Tables of integrals, common and natural logarithms, trigonometric functions, and exponential and hyperbolic functions, are included in the text. Answers to most of the exercises can be found at the end of the book.

An abundance of graded exercises, judicious exposition of mathematical theory, excellent illustrations and drawings, and clear typography on fine quality paper, makes this text a worthy addition to the many books presently available on the subjects of analytic geometry and the calculus.—ARTHUR SVOBODA, De Paul University, Chicago, Illinois.

*A Manual for the Slide Rule*, Paul E. Machovina. New York, McGraw Hill-Book Company Inc., 1950. 78 pp., $0.75.

This manual deserves a high place among the various writings on the slide rule. It is excellent from a pedagogical standpoint. Explanations are clear and sufficient without being too detailed. In addition to the usual history and explanation of construction, Machovina presents classifications of slide rules, and helpful suggestions for the selection and care of the instrument.

Extreme care is taken to show the meaning behind the basic processes of multiplication and division. For example, Gunther's method of adding logarithms by means of dividers is illustrated by diagrams, and the interchange of indices is made meaningful by sketching imaginary continuations of the C and D scales. Explanation is given for each scale commonly found on the log log duplex slide rule. These include the folded inverted, and log log scales in addition to the standard A, B, C, D, S, T, etc. scales.

Memorization of rules is minimized with emphasis being placed on understanding and application. Included in the manual are eight detachable problem sheets, each containing a number of practical exercises. The variation in difficulty should provide incentive for students at either the secondary or college level.—RANDOLPH S. GARDNER, New York State College for Teachers, Albany, New York.

*Practical Use of the Slide Rule* (Reprint), Calvin C. Bishop. New York, Barnes and Noble, Inc., 1950. vii+147 pp., $.75.

This book is one of the inexpensive paper bound College Outline Series. Intended primarily for the college student, it is written in a clear and concise manner, and yet contains a thoroughness which is rare for a book of such small size.

The book is divided into two parts. Part I contains the explanations for operations with the basic scales of any slide rule, namely the *A, B, C, D, K, CI*, logarithmic and trigonometric scales. These are well illustrated with diagrams. Some readers might desire a more detailed ex-

planation of the method of reading the scales. Ordinarily considered a matter of primary importance, this is relegated to a very brief discussion, without diagrams, in the author's introduction. Considerable space is given to another point of difficulty however. The author's system of locating the decimal point (a variation of the standard number method) is presented in detail.

Part II consists of charts and sample settings for five well known slide rules including the log log duplex for trig and vector rules, the "Hemmi" log log vector, and the Langsner Industrial slide rules.

A feature of the book is the variety of excellent drill exercises and applied problems given with answers. They cover many of the problems of geometry, trigonometry, mechanics, chemistry, electrical engineering and thermodynamics.—RANDOLPH S. GARDNER, New York State College for Teachers, Albany, New York.

*Mathematics of Finance*, Albert E. May. New York, American Book Company, 1951. viii+264 pp., $3.00.

This text includes the topics in business mathematics and life insurance usually found in a book under this title. The book as a whole, and the chapters as well, are organized in a most effective manner. The manuscript has been carefully prepared. The printed page, with a few exceptions, is attractive and easy to read.

It is unfortunate that the explanatory notes have been placed in such small type. This applies also to the combinatorial symbols $a_{n/i}$, and $s_{n/i}$.

Throughout the book the author has done an excellent job in emphasizing the importance of understanding the basic principles involved. To this end he has presented well-thought-out discussions, offered many illustrations, and has kept the formulas needed to a minimum.

The tables are up-to-date and excellent. Answers to odd problems are given.

Anyone wanting a good book in the mathematics of finance should not fail to see this book.—WILLIAM A. GAGER, University of Florida, Gainesville, Florida.

*Mathematics Essential for Elementary Statistics* (Revised ed.), Helen M. Walker. New York, Henry Holt and Co., 1951. xiii+381 pp., $2.75.

In the words of the author, "I have tried to write a small book . . . enabling adults quickly to refresh their knowledge of elementary algebra essential to the study of statistical method but that will also provide an introduction to mathematical thinking as clear and as simple as the best high school texts but with style and content suited to the adult reader."

The revised edition has been expanded from the original (1934) edition to include more work in arithmetic and enough more work in algebra so that it may be read intelligently by a person who has never had even the rudiments of algebra. The scope of the book is best indicated by the following partial table of contents: Simple operations with common and decimal fractions; short cuts in computations; weights, measures, and solutions; algebraic symbolism; directed numbers; simple functions and their graphs, linear equations; inequalities, ratios and trigonometric functions; linear interpolation; exponents and radicals; logarithms; permutations and combinations; the binomial expansion; the multinomial expansion; equations of the second degree in two variables; degrees of freedom; equations with several variables.

Each chapter is preceded by a preliminary test to enable the reader to determine which, if any, sections of the chapter he needs to study. Exercises are numerous, and at the end of each chapter is a final test. Answers to the tests and exercises are given.

The book is attractively bound and printed. It is truly self-teaching, and fills a real need of those students of elementary statistics and social and natural sciences whose mathematical background is inadequate.—R. A. BEAVER, New York State College for Teachers, Albany, New York.

*Technological Applications of Statistics*, L. H. C. Tippett. New York, John Wiley and Sons, Inc., 1950. ix+189 pp., $3.50.

This book treats of applied statistics in two parts; the routine control of quality, and investigation and experimentation.

The discussion of quality control is clear and concise, with some good applications from British industry. The discussion of bias in sampling is of particular interest.

In part two variance, statistical theory of errors and correlation are discussed with numerous applications of each.

This book is very easy to read, and should appeal to anyone interested in applied statistics, even those with little or no mathematics background.—WILLIAM E. FELLING, Parks College of Aeronautical Technology, East St. Louis, Illinois.

*Elements of Ordinary Differential Equations* Michael Golomb and Merrill Shanks. New York, McGraw-Hill Book Company, Inc., 1950. ix+356 pp., $3.50.

The authors aptly state that this book "is designed to appeal to students majoring in engineering, science or mathematics." Furthermore, "every effort has been made to attain precision in treatment without sacrificing clarity or becoming too involved with questions of rigor."

There are eight chapters: a good review, geometric fundamentals and techniques for solving first-order equations, second-order equations, linear equations of higher order, algebra of operators, numerical solution and solution in power series.

Applications and miscellaneous problems are exceptionally well done and the many welcome hints and references that are given to aid the serious reader are the most outstanding feature of the book.

There is too much material for a one-semester course but the authors have outlined what they think should be given in such a course.

This text would be a happy choice for a mixed class of engineers and liberal art students if each of them possessed a love for mathematical thinking.—SAMUEL F. BIBB, Illinois Institute of Technology, Chicago, Illinois.

*Introduction to Number Theory*, Trygve Nagell. New York, John Wiley and Sons, Inc., 1951. 309 pp., $5.00.

This book contains one of the more enlightened accounts of elementary number theory that is available in English. The greater part of it can be read by anyone with a good background in classical algebra and the requisite patience. The main feature of the book is the remarkable success with which it conveys to a reader, even of modest mathematical attainments, the attitude of mind that prevails in modern number theory.

The first part of the book is concerned primarily with divisibility, congruences and quadratic reciprocity. In the excellent second chapter on the distribution of primes, there is an informal discussion of Riemann's zeta function and his celebrated conjecture. Two chapters are devoted to diophantine analysis, in which connection it is indicated that the problems raised, both in their arithmetical and geometrical interpretations, are of significance for number fields more general than the rationals. The last chapter is devoted to an elementary proof of the Prime Number Theorem, essentially that given by Paul Erdös in 1948.

This book should prove to be a source of strong inspiration to young prospective mathematicians, especially those living in smaller communities where mathematical books are not ordinarily available.—W. E. JENNER, Northwestern University, Evanston, Illinois.

*Introduction to Modern Algebra and Matrix Theory*, O. Schreier and E. Sperner. Translated by Melvin Hausner and Martin Davis from *Einfuhrung in die Analytische Geometrie und Algebra*. New York, Chelsea Publishing Co., 1951, viii+378 pp., $4.95.

This outstanding book forms a good introduction to some of the topics and methods of modern algebra, and appears to be well suited for use as a text in a course of that nature. The treatment of the subject matter is concrete, and frequent geometric illustrations and motivations are given. The concepts of *vector* and *linear transformation* underlie and motivate the discussion throughout. The student is presumed to have an intuitive conception of the real number system and its properties; otherwise the book is nearly self-contained, and each topic is painstakingly developed. Although it is therefore possible for a student with little background to understand it, a certain amount of mathematical maturity would be desirable, in view of the sophistication and general outlook of the book. There are numerous well-chosen exercises, some in illustration and others in extension of the text.

The translation covers volume one and half of volume two of the 1935 German edition (which differs considerably from the 1948–51 edition), omitting the chapter on projective geometry. Summary of contents: $n$-dimensional affine space, vectors, vector spaces, linear spaces, linear equations; volumes and determinants, transformation of coordinates, normal orthogonal systems of vectors, rigid motions, affine transformations; fields, polynomials, complex numbers, "fundamental theorem of algebra"; group theory (basis theorem for Abelian groups); the algebra of linear transformations, calculations with matrices, minimal polynomial, invariant sub-spaces, nullspace, diagonal form, elementary divisors, normal forms.—T. C. HOLYOKE, Northwestern University, Evanston, Illinois.

*First Course in Probability and Statistics*. J. Neyman. New York, Henry Holt and Company, 1950. ix+350 pp., $3.50.

According to the author, "As conceived in this book, the theory of statistics is a section of the theory of probability." Consequently, this book is mainly concerned with the basic concepts of probability and their extension to problems of statistics for which the data are of a probabilistic nature. It follows therefore that little of what is commonly termed "Descriptive Statistics" is included. The applications are mainly in the biological and physical sciences, rather than in the social studies.

Opinions vary as to the wisdom of attempting to teach the basic concepts of the theory of statistics before the student has achieved considerable mathematical maturity. However, the author, who has been intimately connected with the development of modern statistical theory, as well as an inspiring teacher of the subject, has provided here material for a beginning one-semester course with the mathematical prerequisite of only high school algebra. This does not mean that the exposition is nonmathematical; on the contrary, considerable attention is given to the concept and use of mathematical models. However, the essential requirement for mastery of the text is to keep up with the mathematical symbols and concepts as they are carefully and skillfully developed from basic principles to the more advanced phases.

There are five chapters in this book. The first chapter is an introduction and explains the scope of the theory of statistics, the scope of probability, and the relation between the two. Chapter Two is concerned with an elementary presentation of the calculus of probability. The theory developed in the second chapter is applied in Chapter Three to problems in genetics. Random variables and frequency distributions discussed in the fourth chapter gives, on an elementary level, the basic concepts of the "Neyman-Pearson" theory and might be studied profitably by advanced as well as by elementary students of statistical theory.

The author is to be highly complimented for a job well and carefully done and this book will prove very valuable to the conscientious student.—JOSEPH A. PIERCE, Texas Southern University, Houston, Texas.

*Introduction to Algebraic Geometry*, W. Gordon Welchman. New York, Cambridge University Press, 1950. x+349 pp., $4.50.

This is an advanced undergraduate college text to be used as an introduction to the "Algebraic Geometry of the Complex Numbers." The author's aim is to prepare students as rapidly as possible for "the study of configurations, loci, and transformations in space of three, four and five dimensions." His procedure is to develop algebraic and geometric foundations and the notation in the first three chapters, and to consider primarily the theory of conics in the remaining chapters using techniques that may also be used in more advanced work.

The exposition is very well organized and at times elegant. Many of the topics usually considered in Projective Geometry are presented from an algebraic viewpoint as illustrated by the author's development (in order but not consecutively) of the topics: homogeneous coordinates, cross-ratio, involutions, conics, pole and polar, quadrangle, and quadrangular sets consisting of three pairs of an involution. There are no sets of problems and no index.

This book does not use the terminology of abstract algebra. It also differs from many other texts of algebraic geometry in that it is concerned only with "space of one, two and three dimensions, rational systems, linear transformations, curves and surfaces of order two, and the like." Mathematically mature readers will find in this text a well-developed presentation of the techniques under consideration.—B. E. MESERVE, University of Illinois, Urbana, Illinois.

*Vitalizing Secondary Education*, Federal Security Agency, Office of Education, Bulletin 1951, No. 3. 30¢.

This is another in the series of the U. S. Office of Education publications dealing with Life Adjustment Education, one of the programs which the Office of Education is sponsoring.

The bulleting opens with two chapters which sketch the social, economic, and educational changes in the United States from 1890–1945. These changes created the need for attention to a more functional education for "the remaining 60% of students in secondary schools." This is assuming that the needs of the 20% who are afforded a vocational education are adequately met. The principles defining an adequate education for the 60% are presented in a section "What is life adjustment education?" The work of the National Commission on Life Adjustment Education for Youth in promoting this kind of education is described. There follow two chapters which describe various state and local school programs of education which are considered to be in the spirit of life adjustment education.

The committee which prepared this bulletin says frankly, "The basic concepts underlying life adjustment education are not new." And when one reads the definition of this education, the statement is seen to be true. Few would quarrel with the general principles which are enunciated. Educators have accepted them long before the National Commission on Life Adjustment Education was formed and before Dr. C. A. Prosser made his resolution that got the whole program under way. It seems, therefore, unfortunate that a new name is given to an old idea.

The bulletin is useful for learning briefly what is being done to improve education in various state and local schools. There are no specific allusions to mathematics education, so mathematics teachers would read it for general information. Gratifying to the reviewer throughout the entire presentation is the idea that there is no one curricular organization or method of teaching that is a panacea for all our educational ills. The reviewer sensed little attempt to "sell a bill of goods." The bulletin is mainly descriptive rather than exhortatory. Unfortunately, for some reason, the various state and local programs are presented anonymously and piecemeal. Hence, one cannot follow up a particular program that is described. However, a list of the State Departments of Education and/or State Committees, and local schools which submitted materials and described practices related to life adjustment education is given in an Appendix.—K. B. HENDERSON, University of Illinois, Urbana, Illinois.

*Sourcebook on Atomic Energy*, Samuel Glasstone, New York, D. Van Nostrand Company, Inc., 1950. ii+546 pp., $2.90.

This book gives a very comprehensive picture of the development and present status of the subject of atomic energy as far as it can be told. Quotations from the pioneering scientists add much to the feeling of authenticity. The author treats the subjects of atomic structure, radiation, radioactivity, cosmic rays, nuclear energy, new elements and health physics. Equations and diagrams are used only to present ideas. The book presents a well written and very readable reference for those desiring an understanding of atomic science. It is a complete source book on atomic energy.—ROGER M. MORROW, Western Illinois State College, Macomb, Illinois.

*Differential and Integral Calculus*, E. Landau. Translated by Melvin Hausner and Martin Davis from *Einführung in die Differentialrechnung und Integralrechnung*. New York, Chelsea Publishing Co., 1951. 366 pp., $5.00.

This extremely literal translation preserves the exceptional and spacious readability of the original masterpiece. Little background is required, the whole presentation being built up rigorously in a self-contained fashion from the properties of the real number system. Examples

and counter-examples are plentiful (e.g., one concerning an everywhere continuous, nowhere differentiable function). These and other good features make it an excellent reference book in spite of two serious mechanical defects: (a) no index; (b) independent numbering systems for definitions, theorems, and chapters. The book merits serious consideration as a text for advanced calculus, though again there appear to be drawbacks; (a) no problems or other exercises; (b) no geometric applications (e.g., $\pi$ is described only as twice the least positive root of cos $x$, a function treated entirely through series); (c) no material on multiple integrals.

Except for two errors in translation (p. 49, lines 11 and 15; p. 191, line 9, "so that" for "und"), this edition seems carefully prepared. Summary of contents: introduction (including statement of assumptions), limits, function and continuity, derivative, properties of continuous functions, theorem of the mean, Taylor's theorem, indeterminate forms, series, exponential and trigonometric series, partial derivative, inverse function and implicit function, "fundamental theorem of algebra," partial fraction decomposition of rational functions, indefinite integral, techniques of integration, definite integral, integration of infinite series, improper integral, Gamma function, Fourier series.—T. C. Holyoke, Northwestern University, Evanston, Illinois.

*Classical Mechanics*, Herbert Goldstein. Cambridge, Addison-Wesley Press Inc., 1950. xii+399 pp., $7.00.

While requiring no mathematical background other than vector analysis and some advanced calculus, this book introduces the reader to a large number of fundamental ideas in modern physics. The more complicated mathematical tools required to probe the subject are carefully introduced and developed as the need for them arises.

The author begins with a survey of elementary ideas in mechanics of a particle and proceeds to develop Lagrange's equation of motion. Next he introduces the variational principle and proceeds to develop the Lagrange equations from Hamilton's principle. After introducing the subject of the "two body problem," the author develops the concept of the scattering of particles by a fixed center of force.

In the chapter on Kinematics of Rigid Body Motion, the topic of orthogonal transformations is introduced, and then applied in the transformations from a Cartesian system to a set of Euler axes. Tensors are introduced in the chapter on The Rigid Body Equations of Motion.

An entire chapter is devoted to the subject of special relativity with emphasis on its relations to classical mechanics. This is followed by chapters on The Hamilton Equations of Motion, Canonical Transformations, Hamilton-Jacobi Theory, and Small Oscillations. The final chapter is an Introduction to the Lagrangian and Hamiltonian Formulations for Continuous Systems and Fields.

One extremely valuable feature of this work is the list of suggested references at the end of each chapter. In these, the author's pertinent comments enable the student to find exactly the reference material he needs in order to keep abreast of the subject.—E. W. Banhagel, Detroit, Michigan.

*Les Grands Courants de la Pensée Mathématique*, Présentés per F. Le Lionnais. Marseille, Cahiers du Sud, 1948. 533 pp., 840 fr. ($5.00)

It is difficult to begin to write a review of this volume. The material contained therein would overwhelm anyone.

Actually, this book is a symposium. It contains fifty papers by luminaries in various fields of mathematics. To mention just a few, Borel, Frechet, Deltheil, Andre Saint Lague, Montel, Cartan, Dieudonne, Perrin, Godeaux, and N. Bourbaki (a *nome de plume* for a collection of authors who remain anonymous individually).

The aim of this book is to present in as simple a language and in as elementary a treatment as possible a complete and thorough survey of the domain of mathematics, the mathematics as it is known to mathematicians *now*. The authors who were selected for this purpose are specialists in their own fields, and they presented their case in short chapters with the aim of reaching the *educated layman*. This they have accomplished with an unexcelled literary mastery (the French language is singularly suitable for such purposes). One recalls, while reading this book, the style and lucidity of the expositions of Henri Poincaré.

The book is divided into three parts. This division represents the editorial philosophy of M. F. Le Lionnais. When he conceived this idea (in 1942, while a part of France was under the German occupation) he set as its aim the assessment of mathematics in relation to the subject itself and to civilization. Thus, he decided that the subject must be examined in terms of its structure, its method, its past, present and future, its implications and influences on human thought, its pedagogical and psychological phases, its relation to philosophy, its association with science, technology and the arts.

Each and every individual chapter and section can be read without any specific need for referring to some other chapter and section. Thus, this book, although it is cumulative in nature, structure and form, can be read easily and in stages. In totality, however, this book represents one of the best available sources for gaining a perfect bird's-eye view of the entire field of mathematics. As a survey it probably has not peers at present.

And here is a suggestion. Not many of the mathematics teachers can read French. A translation of this book (even if this is done by individual chapters) would place in the hands of the teachers something which would be extremely valuable.—A. Bakst, Flushing, New York.

# Program
# The Thirteenth Christmas Meeting
# The National Council of Teachers of Mathematics

**Hotel Lincoln, Lincoln, Nebraska**
**December 29, 30, 31, 1952**

**Host Organization:** Nebraska Section of the National Council of Teachers of Mathematics and Teachers College, University of Nebraska
**Convention Theme:** Mathematics Appropriate for Students Today

MONDAY, DECEMBER 29, 1952

1:30 P.M.–9:00 P.M. **Registration**—Mezzanine Floor

1:30 P.M.–5:00 P.M. **Meeting of the Board of Directors**—Parlor C

2:00 P.M.–5:30 P.M. Sight-seeing Trip Through Nebraska's Beautiful State Capitol and the University of Nebraska Museum

7:30 P.M. **Entertainment**—Ballroom
Men's Style Show and Square Dancing

8:00 P.M.–10:00 P.M. **Meeting of the Board of Directors**—Parlor C

TUESDAY, DECEMBER 30, 1952

8:00 A.M.–5:00 P.M. **Registration**—Messanine Floor

9:00 A.M.–10:15 A.M. **General Session**—Ballroom
Presiding: JOHN R. MAYOR, President of the National Council of Teachers of Mathematics, University of Wisconsin, Madison, Wisconsin
*Our Mathematical Resources—How Can We Use Them?* MARTHA HILDEBRANDT, Proviso Township High School, Maywood, Illinois

10:30 A.M.–12:15 P.M. **Elementary Section**—English Room
Presiding: ETHEL LONG, Lincoln Public Schools, Lincoln, Nebraska
*Desirable Mathematical Background of Elementary Teachers*, BESS M. TOWNSEND, North Texas State College, Denton, Texas
*Using Mathematical Materials More Effectively in Teaching Primary Arithmetic*, RUTH GUTHRIE, Pulaski County Special School District, Little Rock, Arkansas
*How to Teach Common Denominators Meaningfully*, IRENE SAUBLE, Detroit Public Schools, Detroit, Michigan

10:30 A.M.–12:15 P.M. **Junior High School Section**—Garden Room. This section is sponsored by the Colorado Council of Teachers of Mathematics.
Presiding: J. HARVEY LITTRELL, Hastings Public Schools, Hastings, Nebraska
*Modern Materials and Methods for Junior High School*, WILLIAM INGRAM, East High School, Denver, Colorado
*Mathematical Visual Aids for the Elementary School with Implications for Their Use in Junior High School Teaching*, RUTH TUTTLE, Palmer School, Denver, Colorado
*A New Approach to Mathematics for the Consumer*, WILLIAM H. KRUSE, Grant Junior High School, Denver, Colorado

10:30 A.M.–12:15 P.M. **High School Section**—Terrace Room
Symposium—*What Mathematics Should Be Required of High School Graduates?*
Chairman: FLOYD A. MILLER, State Department of Public Instruction, Nebraska
Participants: HARRY W. CHARLESWORTH, East High School, Denver, Colorado; MAURICE L. HARTUNG, University of Chicago, Chicago, Illinois; JULIUS H. HLAVATY, Bronx High

School of Science, New York City; W. I. LAYTON, Stephen F. Austin State College, Nacogdoches, Texas; GALEN SAYLOR, University of Nebraska, Lincoln, Nebraska

10:30 A.M.–12:15 P.M. **College and Teacher Education Section**—Chinese Room. This section is sponsored in part by the Arkansas Council of Teachers of Mathematics.

Presiding: JOHN ABERNETHY, Arkansas Polytechnic College, Russellville, Arkansas

*Some Classroom Techniques in the Teaching of Elementary College Mathematics*, KENNETH W. WEGNER, Carlton College, Northfield, Minnesota

*Stressing Mathematical Concepts Which Arise in Non-Mathematical Subjects*, GARLAND D. KYLE, Agricultural, Mechanical and Normal College, Pine Bluff, Arkansas

*The Englishman's Mathematics in the Eighteenth Century*, N. A. GOLDSMITH, Henderson State Teachers College, Arkadelphia, Arkansas

10:30 A.M.–12:15 P.M. **Approximate Number Section**—Arbor Room

Presiding: JOSEPHINE E. WIBLE, Lincoln High School, Lincoln, Nebraska

*Computation with Approximate Numbers*, CECIL B. READ, University of Wichita, Wichita, Kansas

*What Proficiency in the Use of the Slide Rule Is Expected of Engineering Students?* DONALD C. HAACK, Engineering School, University of Nebraska, Lincoln, Nebraska

2:00 P.M.–3:45 P.M. **Elementary and Junior High School Section**—Garden Room

Presiding: L. W. LAVENGOOD, Tulsa Public Schools, Tulsa, Oklahoma

Topic: *Problem Solving for Today's Children*

Leader: HENRY VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa

Assistants: ALICE ROSE CARR, Ball State Teachers College, Muncie, Indiana; JACK HALL, Kansas State Teachers College, Pittsburg, Kansas; and INEZ PETERSON, Sioux City Public Schools, Sioux City, Iowa

2:00 P.M.–3:45 P.M. **High School Section**—Terrace Room

Presiding: MARIE S. WILCOX, Washington High School, Indianapolis, Indiana

*Geometry for All Laymen*, HAROLD FAWCETT, Ohio State University, Columbus, Ohio

*The Role of Class Procedures in Teaching Geometry*, EUGENE SMITH, Laboratory School, Ohio State University, Columbus, Ohio

*Combined Course of Plane and Solid Geometry*, KATHLEEN O'DONNELL, Laboratory School, University of Kansas, Lawrence, Kansas

2:00 P.M.–3:45 P.M. **General Session**—Arbor Room

Chairman: DONOVAN A. JOHNSON, University of Minnesota High School, Minneapolis, Minnesota

Panel Discussion: *The Evaluation of Multi-Sensory Aids*

Participants: GEORGE DAHL, New Ulm High School, New Ulm, Minnesota; MAURICE L. HARTUNG, University of Chicago, Chicago, Illinois; JOHN MOLDSTAD, Audio-Visual Department, Indiana University, Bloomington, Indiana

2:00 P.M.–3:45 P.M. **Teacher Education Section**—Chinese Room

Presiding: CECIL B. READ, University of Wichita, Wichita, Kansas

*A Student's Syllabus for Methods of Teaching Secondary School Mathematics*, KATHARINE HOSKINS, State Teachers College, Minot, North Dakota

*Is the Program in Mathematics Offered by Colleges and Universities Adequate to Meet the Needs of Secondary Mathematics Teachers?*

(a) *From the Standpoint of Undergraduate Study*, E. W. HELLMICH, Northern Illinois State Teachers

College, DeKalb, Illinois

(b) *From the Standpoint of the Masters Degree*, H. G. AYRE, Western Illinois State College, Macomb, Illinois

2:00 P.M.–3:45 P.M. **The Small High School Section**—English Room

Chairman: L. M. LARSEN, Nebraska State Teachers College, Kearney, Nebraska

Panel Discussion: *Mathematical Problems of the Small High School*

Participants: H. VERNON PRICE, State University of Iowa, Iowa City, Iowa; ANNA CARESS, Stapleton High School, Stapleton, Nebraska; EMILY WILSON, Nelson High School, Nelson, Nebraska; C. L. RETELSDORF, Valley Public School, Valley, Nebraska and D. O. PATTERSON, Colorado State College of Education, Greeley, Colorado

4:00 P.M.–5:00 P.M. **Showing of Films and Filmstrips**

Elementary—English Room

Secondary—Garden Room

4:00 P.M.–5:00 P.M. **Gifted Student Section**—Parlor C

Presiding: HENRIETTA COURTRIGHT, Junior College, Arkansas City, Kansas

*Using What We Know About Children in Planning Arithmetic Experiences for the Gifted Child*, IRENE LARSON, Green Bay Public Schools, Green Bay, Wisconsin

*Mathematical Curriculum and Extra-Curriculum for the Gifted*, JULIUS H. HLAVATY, Bronx High School of Science, New York City

4:00 P.M.–5:00 P.M. **Algebra Section**—Chinese Room

Chairman: ANTON RICHERT, Wichita High School East, Wichita, Kansas

Panel Discussion: *Advanced Alegbra, One or Two Semesters—Organization of Topics*

Participants: A. W. GOODWIN, East High School, Des Moines, Iowa; ONA KRAFT, Collinwood High School, Cleveland, Ohio; L. W. LAVENGOOD, Tulsa Public Schools, Tulsa, Oklahoma; GRACE MCMAHON, Lincoln High School, Lincoln, Nebraska; ELIZABETH MINCKEMEYER, Paseo High School, Kansas City, Missouri; and MARTHA RAYHILL, Liberty High School, Lawrence, Kansas.

4:00 P.M.–5:00 P.M. **College Section**—Arbor Room

Presiding: ADRIEN L. HESS, Montana State College, Bozeman, Montana

*Space Travel*, WALLACE HERBERT, Louisiana Polytechnic Institute, Ruston, Louisiana

*Rapid Calculation*, ELMER HAM, Arkansas Polytechnic College, Russellville, Arkansas

4:00 P.M.–5:30 P.M. **Affiliated Groups Section**—Terrace Room

Chairman: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey

*Technique for Introducing Decimals*, ALICE ROSE CARR, Ball State Teachers College, Muncie, Indiana

*Learning Devices for the Junior High School Student*, VERYL SCHULT, Public Schools, Washington, D. C.

*Building an Understanding of Number Concepts*, DONOVAN A. JOHNSON, University High School, University of Minnesota, Minneapolis, Minnesota

*My Six Teaching Aids for Plane Geometry*, AMELIA RICHARDSON, McKeesport High School, McKeesport, Pennsylvania.

Topic to be announced, W. V. UNRUH, Indian Mission School, Merriam, Kansas

6:15 P.M. **Banquet**—Ballroom

Presiding: MILTON W. BECKMANN, Teachers College, University of Nebraska, Lincoln, Nebraska

*The Place of Mathematics in the Curriculum*, REUBEN G. GUSTAVSON, Chancellor of the University of Nebraska, Lincoln, Nebraska

WEDNESDAY, DECEMBER 31

8:00 A.M.–9:00 A.M. **Registration**—Mezza-

nine Floor

8:00 A.M.–9:00 A.M. **Showing of Films and Filmstrips**

Elementary—English Room

Secondary—Garden Room

9:00 A.M.–10:15 A.M. **General Session**—Ballroom

Presiding: JAMES H. ZANT, Oklahoma A. and M. College, Stillwater, Oklahoma

*The Importance of Learning*, HOWARD F. FEHR, Teachers College, Columbia University, New York City

10:30 A.M.–12:15 P.M. **Elementary Section**—English Room

Presiding: D. O. PATTERSON, Colorado State College of Education, Greeley, Colorado

*Correlating Arithmetic With Other School Subjects*, THEODORA NELSON, State Teachers College, Kearney, Nebraska

*Making Mathematics Function in the Total Elementary Curriculum*. CALVIN REED, University of Nebraska, Lincoln, Nebraska

10:30 A.M.–12:15 P.M. **General Mathematics Section**—Garden Room

Presiding: AGNES HERBERT, Clifton Park Junior High School, Baltimore, Maryland

*Effective Techniques and Devices for General Mathematics*, ALICE M. HACH, Slauson Junior High School, Ann Arbor, Michigan

*Essentials of Mathematics for Juniors and Seniors*, MARGARET JOSEPH, Shorewood High School, Milwaukee, Wisconsin

10:30 A.M.–12:15 P.M. **Geometry Section**—Chinese Room

Presiding: VIRGINIA LEE PRATT, Central High School, Omaha, Nebraska

*Geometry Comes Alive*, NINA OLIVER, Ardmore, Oklahoma

*Field Work for High School Geometry*, LAUREN G. WOODBY, State Teachers College, Mankato, Minnesota

10:30 A.M.–12:15 P.M. **College Section**—Arbor Room

Presiding: ORLANDO C. KREIDER, Iowa State College, Ames, Iowa

*Adjusting First Year College Mathematics to Meet the Needs of Special Students*, ADRIEN L. HESS, Montana State College, Bozeman, Montana

*Is It Desirable to Have Special Sections for Engineers, Agricultural Majors, Pre-Medics, etc.?* JOHN ABERNETHY, Arkansas Polytechnic College, Russellville, Arkansas

*New Emphases in the Calculus*, SISTER MARY FELICE, Mount Mary College, Milwaukee, Wisconsin

10:30 A.M.–12:15 P.M. **General Session**—Ballroom

Symposium of Tuesday continued with *Emphasis on the Preparation of the College Bound Student*

Presiding: H. W. CHARLESWORTH, East High School, Denver, Colorado

*New Trends in College Entrance Requirements*, MARTHA PETERSON, University of Kansas, Lawrence, Kansas

*The Mathematical Needs of College Preparatory Students*, BRUCE E. MESERVE, University of Illinois, Urbana, Illinois

*Bridging the Gap Between High School and College Mathematics*, W. L. WILLIAMS, University of South Carolina, Columbia, South Carolina

12:30 P.M. **Closing Luncheon**—Terrace Room

Presiding: DUANE PERRY, President of the Nebraska Section of the National Council of Teachers of Mathematics, Omaha Public Schools, Omaha, Nebraska

*Insurance Looks Ahead*, E. J. FAULKNER, President, Woodmen Accident Company, Lincoln, Nebraska

## PROGRAM COMMITTEE

**Chairman—Lucy E. Hall, Wichita, Kansas; H. G. Ayre, Macomb, Illinois; Milton Beckmann, Lincoln, Nebraska; H. W. Charlesworth, Denver, Colorado; Mary Lee Foster, Arkadelphia, Arkansas; E. H. C. Hildebrandt, Evanston, Illinois; Lottchen Lipp Hunter, Wichita, Kansas; Donovan Johnson, Minneapolis, Minnesota; Phillip S. Jones, Ann Arbor, Michigan; John R. Mayor, Madison, Wisconsin;**

Kenneth Nickel, Wichita, Kansas; G. Baley Price, Lawrence, Kansas; Cecil B. Read, Wichita, Kansas; Mary Rogers, Westfield, New Jersey; Burnett Severson, Denver, Colorado.

## ANNOUNCEMENTS

*Registrations*

The registration fee is fifty cents for members of the National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elementary schools. The fee for non-members and visitors is $1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press, and commerical exhibitors are not charged the registration fee but should register. You are urged to register in advance, but, if you do, please check in at the Registration Desk at your arrival at the meeting. Use the Advance Registration and Reservation Form supplied herewith. Hotel Lincoln, Ninth and P Streets is headquarters for the convention. Registration headquarters will be on the mezzanine floor of the Hotel Lincoln.

*Hotel Reservations*

In all hotels, all rooms have complete accommodations except as indicated below. There is a charge of $2.50 extra for a third person in a room. Reservations for hotel rooms should be sent directly to the hotel.

*Banquet and Luncheon Reservations*

Reservations for the banquet on Tuesday and the luncheon on Wednesday should be made in advance. Requests should be accompanied with check or money order. All orders received before December 15 will be acknowledged. Banquet, $2.50; Luncheon, $2.00. Use the Advance Registration and Reservation Form shown on page 491.

*Sight-Seeing Trip Through Nebraska's Beautiful State Capitol and the University of Nebraska Museum*

This trip will leave Hotel Lincoln, Monday, December 29, at 2:30 P.M. No charge.

*Supplies and Equipment*

Speakers and other participants on the program who need blackboards, projection equipment or other materials should communicate with the local chairman, Dr. Milton Beckmann, The University of Nebraska, Lincoln, before December 10.

*Exhibits*

Commercial exhibits of textbooks and teaching aids will be on the mezzanine floor of Hotel Lincoln. Inquiries for exhibit space should be addressed to Mr. Calvin Reed, The University of Nebraska, Lincoln. Teachers are invited to bring exhibits of classroom work. Inquiries

LINCOLN HOTELS AND RATES

| | Single | Double | Twin |
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| Hotel Lincoln | $3.25* | $5.75* | — |
| 9th and P Street | 3.75 | 6.25–7.00 | $7.00– 8.00 |
| Hotel Capitol | 3.50** | 6.00** | — |
| 11th and P Street | 4.00–5.50 | 6.50–8.50 | 8.50– 9.25 |
| Hotel Cornhusker | 3.50** | 6.00** | 6.00** |
| 13th and M Street | 4.25–5.00 | 6.75–7.50 | 9.00–11.00 |
| Hotel Lindel | 2.00–2.75* | 3.50* | 4.50* |
| 13th and M Street | 2.95–3.75 | 4.95–up | 6.00–up |

* Lavatory only.
** No bath.

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Fill out completely and mail to John Killeen, 3800 H Street, Lincoln, Nebraska before December 10, 1952.

Mr., Miss, Mrs. _____
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Registering as member of N.C.T.M. _____; M.A.A. _____; Student _____;

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Members of families and friends of those attending the meetings are welcome to all social and recreational events.

Registration fee: $0.50 or $1.50 Amount $_____

_____ Banquet Reservations at $2.50 each _____

_____ Luncheon Reservations at $2.00 each _____

Total $_____

Make checks payable to Mr. Henry Cox, General Treasurer.

regarding such should be addressed to Miss Maude Holden, Ord, Nebraska.

*Film and Filmstrips*

Mathematics films and filmstrips will be shown as indicated in the program in the English Room and Garden Room. Requests or suggestions regarding films should be addressed to Mrs. Merle Simpson, Extension Department, The University of Nebraska, Lincoln.

*Refunds on Reservations*

No ticket refunds will be made later than three hours prior to the function for which the reservation was made.

*Transportation*

The city of Lincoln may be reached by United Air Lines and Mid West Air Lines; by the Union Pacific Railroad, Chicago, Burlington and Quincy Railroad, Rock Island Railroad, and Chicago and North Western Railroad; by Greyhound Bus Line, American Bus Line, Continental Trailway, Burlington Trailways, and Interstate Transit Line. Hotel Lincoln is two blocks from the Union Station.

## Mathematical Miscellanea

(*Continued from page 475*)

higher level, there are pictures to be developed by combining prescribed parts of curves whose equations are given in rectangular coordinates.

This note extends these pleasant notions a little farther. The diagrams (Fig. 4) depict the outlines of several well-known objects. The polar equations of the curves in question are:

| | | |
|---|---|---|
| A | Lemon | $r=(3+2\sin\theta)^{\sin\theta}$ |
| B | Sock | $r=2(3+2\sin\theta)^{\sin\theta\cos\theta}$ |
| C | Apple | $r=3-2\sin\theta$ |
| D | Gourd | $r=4\theta^{\sin\theta}$ |
| E | Badge | $r=5(3+2\sin\theta)^{\sin^2\theta\cos^2\theta}$ |
| F | Baby Chick | $r=\theta^{\sin\theta\cos\theta}$. |

D. S. Davis
Virginia Polytechnic Institute
Blacksburg, Virginia

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